Regression meets time series

I begin with variations on the theme

$$y_t = \alpha + \beta x_t + \varepsilon_t \; : \; \varepsilon_t \sim IN(0, \sigma^2)$$

where y and x are time series.

Time series analysis has its own terminology often reflecting its roots in physics and engineering. Thus a sequence of independent identically distributed random variables is called **white noise**. The error specification

$$\varepsilon_t \sim IN(0, \sigma^2).$$

is called normal white noise.

We have already seen some regression models for a **single** time series including

- The linear trend model

  $$y_t = \alpha + \beta t + \varepsilon_t \; : \; \varepsilon_t \sim IN(0, \sigma^2)$$

  where the mean Ey_t changes over time ($Ey_t = \alpha + \beta t$).
- The seasonal dummies model

$$y_t = \alpha + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + \varepsilon_t \; : \; \varepsilon_t \sim IN(0, \sigma$$

also with a mean changing over time, taking a different value for each quarter.

Time series analysis also offers many regression models for bi- or multivariate data with distinctive regressors and/or error structures.

Dynamic regressors

In the **distributed lag model**

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \varepsilon_t \; : \; \varepsilon_t \sim IN(0, \sigma^2)$$

current y reflects past as well as present x: so that part of the effect of x is realised immediately and part after a lag.

In the **autoregressive model** the right hand side variable is the dependent variable lagged,

$$y_t = \rho y_{t-1} + \varepsilon_t \; : \; \varepsilon_t \sim IN(0, \sigma^2).$$

These models can be combined into an **autoregressive distributed lag model**

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \gamma y_{t-1} + \varepsilon_t.$$

I have written the models with first order lags but there are analogous forms with higher order lags.

Thus we might be interested in

$$y_t = \alpha + \beta_0 x_t + \ldots + \beta_4 x_{t-4} + \gamma_1 y_{t-1} + \ldots + \gamma_4 y_{t-4} + \varepsilon_t.$$

These models are estimated by ordinary least squares as there are no simultaneity complications.

To see why not recall how in the autoregressive model y is built out of current and past errors

$$\begin{aligned} y_t &= \rho y_{t-1} + \varepsilon_t \\ &= \rho^t y_0 + \varepsilon_t + \rho \varepsilon_{t-1} + \ldots + \rho^{t-1} \varepsilon_1 \end{aligned}$$

While y_{t-1} is built from $\varepsilon_1, \ldots, \varepsilon_{t-1}$ it is independent of ε_t provided ε_t is independent of earlier errors. So regressing y_t on y_{t-1} does not introduce any simultaneity problem.

Fancy errors

Another way to extend the basic regression model is to elaborate the error structure. I describe two ways proposed by time series analysts.

Autoregressive errors

We have already seen autoregressive

errors:

$$y_t = \alpha + \beta x_t + \varepsilon_t$$
$$\varepsilon_t = \rho \varepsilon_{t-1} + u_t \;:\; u_t \sim IN(0, \sigma^2).$$

The idea is that this period's shock is correlated with last period's shock.

OLS does **not** take into account the correlation between ε_t and ε_{t-1} and so loses efficiency. The equation is better estimated by a form of generalised least squares.

When $\rho = 0$ the model reduces to the basic regression model with white noise errors. The Durbin-Watson test produced by EViews is a test of $\rho = 0$.

The DW test uses the least squares residuals and considers the correlation between successive residuals. Because of the way the statistic is constructed a value of 2 corresponds to the case $\rho = 0$.

The DW is a diagnostic test and EViews performs it whenever it does a least squares regression–even for cross-section data where the autoregressive error specification is not usually relevant.

If the regression passes the test (i.e. the hypothesis $\rho = 0$ is **not** rejected) the researcher is happy.

If the hypothesis $\rho = 0$ is **rejected** the researcher will rethink the dynamic part of the model.

One explanation for an adverse DW result is that the observations are actually generated from

$$y_t = \alpha + \beta x_t + \varepsilon_t$$

$$\varepsilon_t = \rho \varepsilon_{t-1} + u_t$$

with $\rho \neq 0$.

BUT another possibility is that the observations come from an ARDL model like

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \gamma y_{t-1} + \varepsilon_t \; : \; \varepsilon_t \sim IN(0, \sigma^2)$$

and the omission of the variables x_{t-1} and y_{t-1} is responsible for the DW result when regressing y_t on x_t.

Some researchers prefer models where the dynamics are in the systematic part to models where the dynamics are in the error part. Instead of a static model subject to a stream of correlated shocks they envisage a dynamic model with independent shocks.

The separation between systematic and error parts is more apparent than real. The model

$$y_t = \alpha + \beta x_t + \varepsilon_t$$
$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t$$

looks very different from an ARDL but it can be rewritten in a similar form.

We saw earlier that putting

$$y_t = \alpha + \beta x_t + \varepsilon_t$$
$$\rho y_{t-1} = \rho\alpha + \rho\beta x_{t-1} + \rho\varepsilon_{t-1}$$

and subtracting (using $\varepsilon_t - \rho\varepsilon_{t-1} = u_t$) we get

$$y_t - \rho y_{t-1} = \alpha - \rho\alpha + \beta x_t - \rho\beta x_{t-1} + u_t$$
$$\Rightarrow y_t = \rho y_{t-1} + \beta x_t - \rho\beta x_{t-1} \text{ etc.}$$

This is an ARDL with a restriction binding the coefficients of y_{t-1}, x_t and x_{t-1}.

ARCH models

The basic regression specification included the assumption that the error variance is constant across observations–homoscedasticity.

Financial time series often exhibit time varying volatility and econometricians have developed ways of modelling it.

A popular specification is the ARCH (autoregressive conditional heteroscedasticity) model

$$y_t = \alpha + \beta x_t + \varepsilon_t$$

$$\sigma_t^2 = \omega + \gamma^2 \varepsilon_{t-1}$$

Here σ_t^2 the one period ahead forecast variance (called the conditional variance) which depends on the "news about volatility" in the preceding time period.

There is a generalised ARCH with

$$\sigma_t^2 = \omega + \gamma^2 \varepsilon_{t-1} + \delta \sigma_{t-1}^2.$$

In 2003 Rob Engle received the Nobel prize for work on these models:

http://www.nobelprize.org/nobel_prizes/economics/laureates

EViews has a suite of ARCH methods under Equation Estimation.