Index numbers

It is often desired to replace a collection of numbers by a single one–an **index number** or **index**–to give an overall impression of the numbers.

Sometimes the index can be rigorously justified and sometimes not. In Example 1 the index follows from arithmetic, in Example 2 index construction is a much more speculative activity while in Example 3 there are principles underlying the choice of index–but a variety of principles.

Examples

1. **Aggregate population growth**. Diagram 2 includes population values for groups of countries such as Western Europe and Latin America. The values for countries are built up from figures for smaller entities, states or provinces. In counting heads there are straightforward part/whole relationships.

Suppose we have growth rates for the provinces of a country, how do we combine them to obtain the national growth rate? In this case the definition of the terms implies a rigorous connection between the component values and the overall value.

Suppose there are two provinces and that at time t population is divided between them so

$$N_t = n_t^1 + n_t^2.$$

The provincial growth rates are defined by

$$g_t^i = \frac{n_t^i - n_{t-1}^i}{n_{t-1}^i}, \quad i = 1, 2.$$

The relation between the national growth rate G_t and the provincial rates g_t^i is found as follows

$$\begin{aligned} G_t &= \frac{N_t - N_{t-1}}{N_{t-1}} \\ &= \frac{(n_t^1 + n_t^2) - (n_{t-1}^1 + n_{t-1}^2)}{N_{t-1}} \\ &= \frac{(n_t^1 - n_{t-1}^1) + (n_t^2 - n_{t-1}^2)}{N_{t-1}} \\ &= \frac{n_{t-1}^1 g_t^1 + n_{t-1}^2 g_t^2}{N_{t-1}} = w_{t-1}^1 g_t^1 + w_{t-1}^2 g_t^2 \end{aligned}$$

The national growth rate is a weighted average of the provincial growth rates where the **weights** w_{t-1}^i are the provincial population shares n_{t-1}^i / N_{t-1}.

For the case of p provinces the argument is the same but with population breakdown and overall growth rate so

$$N_t = \sum_{i=1}^{p} n_t^i,$$

$$G_t = \sum_{i=1}^{p} w_{t-1}^i g_t^i.$$

The analysis is very similar to that for crude and age-specific deaths rates given above.

2. **Social development**: the historian Ian Morris (2010: 135ff, 623ff) has produced an index of social development spanning 12,000 years–from 10,000BCE-2,000CE.

Ian Morris (2010) *Why the West Rules–for Now*, Profile Books.
Morris has an online book on the index: see

http://www.ianmorris.org/docs/social-development.pdf

The index has these components: urbanism (measured by the largest settlement in existence), per capita energy consumption, war making capacity, information technology.

Morris calls his analysis "chain saw art" because it is so rough and ready. There is no underlying theory just arguments about the relevance of these dimensions and the availability of data.

3. GDP–**quantity and price indices**. Diagram 1 compares quantities across countries and across time periods. These comparisons are much more complicated than the comparisons of population numbers in diagram 2 and economists have developed a lot of theory relating to them.

Fundamental to Diagram 1 is the notion of **gross domestic product** though what is plotted is the more complicated quantity, “GDP per capita in 1990 Int. GK$.” See the Maddison file.

GDP is a summary measure of an economy’s output in a particular year. It is given by an expression of the form

$$\sum_{n=1}^{N} p_n x_n$$

where there are N commodities (goods or services) produced in quantities $(x_1, \ldots, x_N)$ at prices $(p_1, \ldots, p_N)$.

GDP figures are used for many purposes. GDP/capita is used as a measure of a country's output available for use by its population. The share of GDP originating in each sector shows the industrial structure of the economy.

The official US and UK procedures for calculating GDP are described in

http://www.bea.gov/national/pdf/NIPAch1-4.pdf

http://www.statistics.gov.uk/hub/economy/national-accounts

According to the US BEA Handbook

> GDP is defined as the market value of the final goods and services produced by labor and property located in the United States. Conceptually, this measure can be arrived at by three separate means: as the sum of goods and services sold to final users, as the sum of income payments and other costs incurred in the production of goods and services, and as the sum of the value added at each stage of production. Although these three ways of measuring GDP are conceptually the same, their calculation may not result in identical estimates of GDP because of differences in data sources, timing, and estimation techniques.

The concept was developed in Britain and the US in the 1940s for market economies: what would the **market value** mean in a **pure command economy** in which nothing is traded and a central planner decides how much of everything is to be produced and how much of output is to go to any final user?

There are many problems with measuring GDP–such as the treatment of goods that are not marketed–and Naughton (ch. 6) considers some special problems associated with measuring the GDP of China today and in the recent past.

Naughton (153-4) points out that before the economic reforms the state followed a high-price policy for industrial output leading to an overstatement of industry’s contribution to GDP. In 1978 prices the share of industry in GDP was 48%, agriculture 28% and services 24%, reflecting the low prices given to agricultural goods. The share of industry in 2004 in 2004 prices was 46% when agricultural goods were given greater weight.

Comparisons. Suppose we have GDP for years 1 and 2

$$\sum_{n=1}^{N} p_n^1 x_n^1 \text{ and } \sum_{n=1}^{N} p_n^2 x_n^2$$

both measured in current prices. How do we compare them, how can we say whether and by much output has grown or contracted?

The usual method is to compare the value of the bundle/basket of goods produced in each period at constant prices, which involves calculating sums like $\sum p_n^2 x_n^1$, the cost of the first basket at second period prices, and $\sum p_n^1 x_n^2$, the cost of the second basket at first period prices.

There are two natural ways of measuring growth of GDP which involve using constant prices

$$X_P = \frac{\sum p_n^1 x_n^2}{\sum p_n^1 x_n^1}; \; X_L = \frac{\sum p_n^2 x_n^2}{\sum p_n^2 x_n^1},$$

named after the 19th century statisticians Paasche and Laspeyres. The P index uses

base period prices and the L index the **current period** prices.

If relative prices are very different in the two periods then P and L can produce very different estimates of growth. Referring to China in 1978 and 2004, the relative prices of agricultural and industrial goods changed and this will affect growth rates.

As well as overall measures of output we often need overall measures of prices, or measures of the price level. We can then talk about the rate of inflation.

There are Paasche and Laspeyres price indexes defined so

$$P_P = \frac{\sum p_n^2 x_n^1}{\sum p_n^1 x_n^1}; \; P_L = \frac{\sum p_n^2 x_n^2}{\sum p_n^1 x_n^2}$$

where the prices in the two years are compared using either base period or current period outputs.

By using constant prices we can compare the GDP of a country at different times. Diagram 1 is based on the idea that GDPs of different countries at different times can be compared. The Maddison file gives a value for GDP per capita in the UK in 1830 of 1,749 and for China in 1987 of 1,737.

Suppose the Chinese and the US statistical bureaux produce GDP figures

$$\sum_{n=1}^{N} p_n^C x_n^C \text{ and } \sum_{n=1}^{N} p_n^{US} x_n^{US}$$

in current (and local) prices. How do we compare them?

One possibility is to use the $/RMB exchange rate to make the figures comparable. The exchange rate today is 1 yuan = 0.1569 dollar. So we might compare

$$0.1569 \cdot \sum_{n=1}^{N} p_n^C x_n^C \text{ and } \sum_{n=1}^{N} p_n^{US} x_n^{US}$$

which are both in US dollars.

However the price structure in the two countries is completely different and it is desirable to use common prices to weight quantities in the two countries, comparing say

$$\sum_{n=1}^{N} p_n^w x_n^C \text{ and } \sum_{n=1}^{N} p_n^w x_n^{US}$$

where p_n^w stands for the 'world price' of the n-th good.

The most commonly used method for getting at international valuations is based on work by **Geary** and **Khamis**. The term **purchasing power parity** is also used.

- Maddison uses it, measuring GDP in 1990 International GK dollars.
- The GK method is used in the Penn World Tables.

I won't say any more about individual indexes–of which there are many! Instead I will comment on the way(s) economists think about them.

There is a survey of index number theory in Diewert's, "Index Numbers" originally published in the *New Palgrave Dictionary*.

Diewert expresses the economic **index number problem** as follows: suppose we have price data $(p_1^i, \ldots, p_N^i)$ and quantity data $(x_1^i, \ldots, x_N^i)$ on N commodities pertaining to economic entity i or pertaining to one entity at periods $i = 1,2,\ldots,I.$ The index number problem is to find I numbers P^i and I numbers X^i such that

$$P^i \; X^i = \sum_{n=1}^{N} p_n^i x_n^i \text{ for } i = 1,2,\ldots,I.$$

P^i is the **price index** for period i (or entity i) and X^i is the **quantity index**.

There are two main approaches in economic index number theory: the **test approach** and the **microeconomic theory approach**.

The idea of the **test** approach is to lay down a series of tests which a good estimator should pass. Ideally the test criteria–based on ‘common sense’–do not conflict with each other and only one index number passes all the tests.

Diewert considers 9 test criteria that have been proposed for price indices. (No index satisfies all.)

I will describe one of his tests to give the flavour of the approach.

His **proportionality test** BT2 requires that if all period 2 prices are multiplied by α, then the new price index should equal α times the old price index.

Consider the Paasche with $p_n^2 = \alpha p_n^1$ (all n)

$$P_P = \frac{\sum p_n^2 x_n^1}{\sum p_n^1 x_n^1} = \frac{\sum \alpha p_n^1 x_n^1}{\sum p_n^1 x_n^1}$$

which is α. So P_P passes the test. There are more examples in the exercises.

The sum $\sum p_n^2 x_n^1$ gives the cost of the period

1 basket at period 2 prices.

If outputs of all commodities grew or contracted

If prices were the same in the 2 periods we could use the common prices and compare the weight

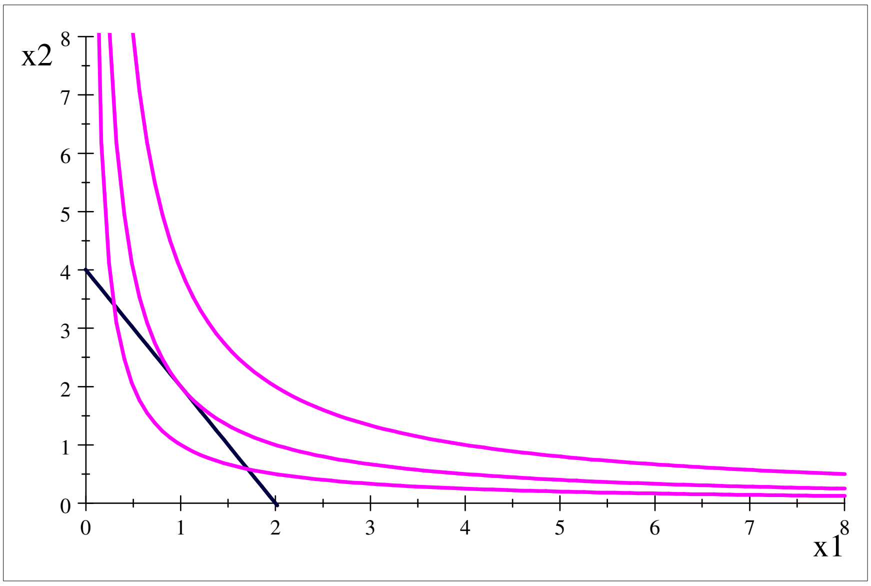
The **microeconomic theory approach** treats the choice of index as part of the neoclassical theory of consumer or producer behaviour.

I will give the flavour of the approach by describing how it treats the choice of price index as a problem in consumer theory.

In consumer theory the consumer has an income which she spends on the different goods on sale. The theory assumes that the consumer has preferences for the goods which can be expressed as a family of indifference curves in which each curve shows the bundle of goods between which the consumer is indifferent.

In the case of two goods the consumer chooses (affordable) values of x_1 and x_2 that give him most satisfaction (put him on the highest indifference curve). The theory assumes that a consumer likes more of a good than less and therefore would rather be on a higher indifference curve than a lower.

The diagram shows this two good case.



Tangency: best the consumer can do

Suppose the prices for the two goods are p_1 and p_2 and the consumer has m to spend. What she can afford is given by the budget constraint (a straight line)

$$p_1 x_1 + p_2 x_2 = m$$

in the diagram $2x_1 + x_2 = 4$

or $x_2 = 4 - 2x_1$

The straight line will be tangential to only one of the indifference curves and that point of tangency represents the **best** the consumer can do when choosing between different bundles of goods.

Economists' consumer theory is largely about the ways purchasers react to changes in prices and income.

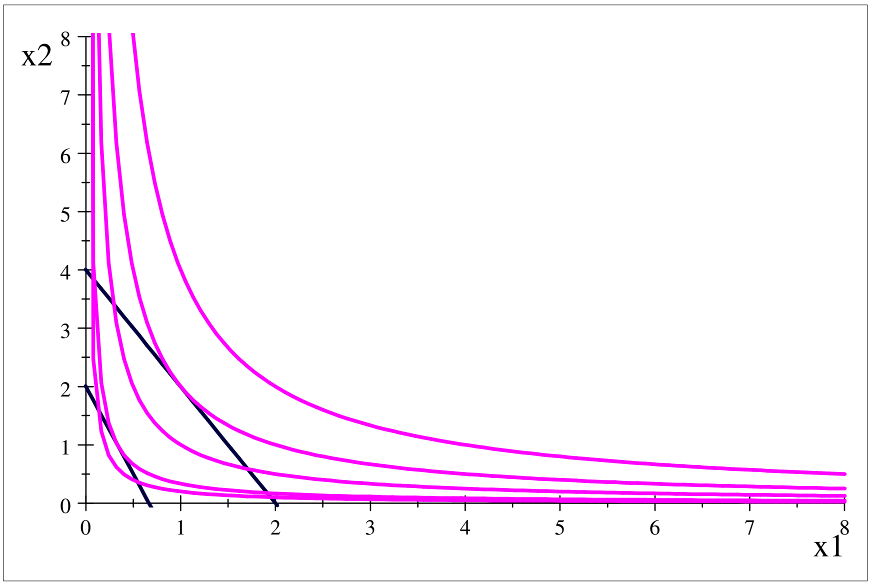
Suppose the price of each good rises–the first trebles, the second doubles–with income **unchanged**. The new budget line is

$$6x_1 + 2x_2 = 4$$

or $x_2 = 2 - 3x_1$.

The **real income** or **purchasing power** of the consumer–what she can afford to buy–is less. Also the **relative** prices of the goods has changed and so she needs to reconsider the proportions she buys of the goods.

Re-drawing the diagram with the new budget line we see that the consumer–being poorer–buys less of each good but also moves away from good 1 which has become relatively dearer to good 2. Although she is doing as well as she can in the circumstances–she is at a tangency point–she is on a lower indifference curve and is worse off.



Prices higher, income unchanged

The price index problem

In the new situation prices are higher. By how much must income increase to leave the consumer just as well off as she was before–i.e. on the same indifference curve? This is a practically important problem e.g. in **indexing** pensions.

If both prices had doubled, a doubling of the pension (income) would leave everything unchanged. If each $p_n^2 = 2p_n$ then

$$2p_1x_1 + 2p_2x_2 = 2m$$

$$\Rightarrow p_1x_1 + p_2x_2 = m$$

There is perfect compensation!

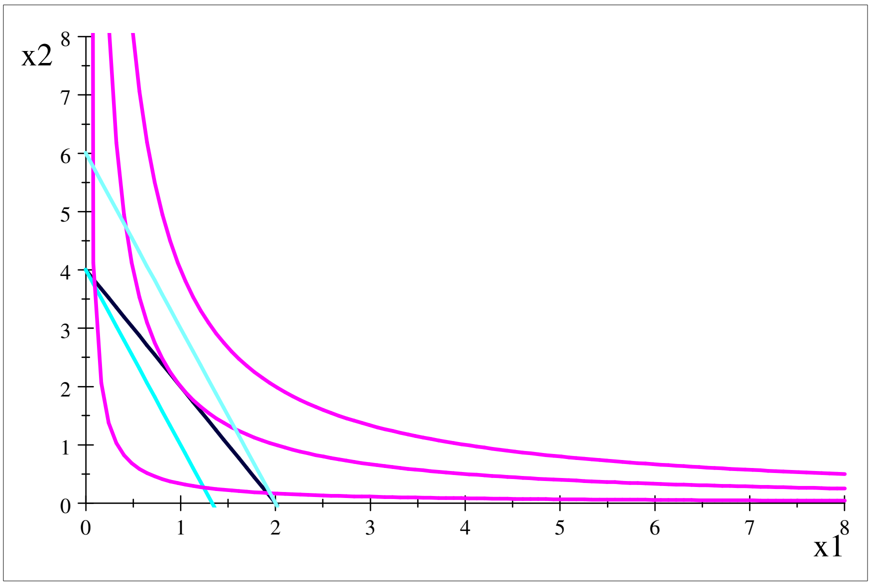
In my example one price had doubled and one had trebled. If the pension m is doubled (4 becomes 8), pensioners are under-compensated for inflation, if it is trebled (4 becomes 12) they are over-compensated. The two 'indexing' schemes

$$6x_1 + 2x_2 = 8$$
$$\text{or } x_2 = 4 - 3x_1.$$

and

$$6x_1 + 2x_2 = 12$$
$$\text{or } x_2 = 6 - 3x_1.$$

are illustrated on the diagram. E.g. it can be seen that with a pension of 8 it is not possible to get on as high an indifference curve as the original one



Two ‘indexing’ schemes

Now consider indexing using the Paasche index

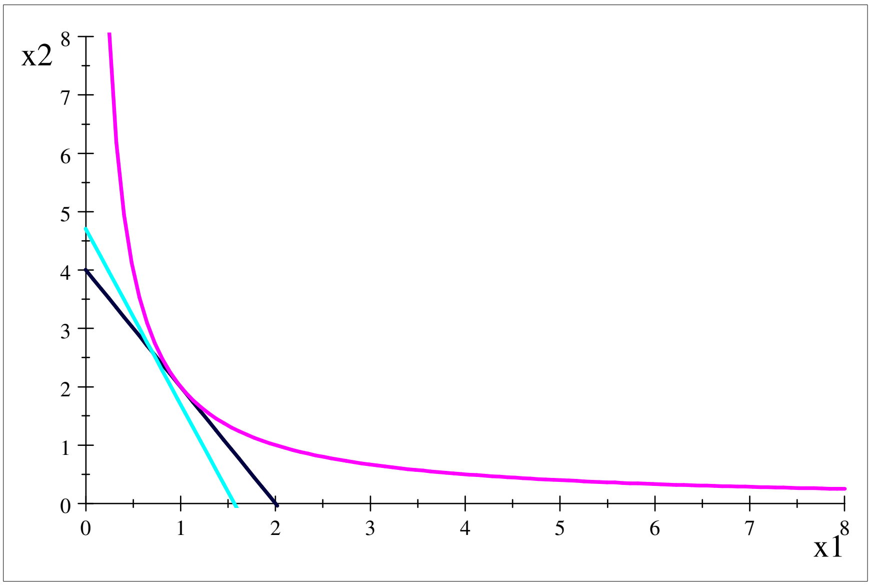
$$P_P = \frac{\sum p_n^2 x_n^1}{\sum p_n^1 x_n^1}.$$

i.e. multiplying m by P_P.

With income of $P_P \cdot m$ the consumer can afford to buy the old consumption bundle at the new prices and so cannot be worse off. She may be better off because she is able to get on to a higher indifference curve.

So indexing the pension by P_P would **overcompensate** the pensioner for inflation.

A perfect indexing scheme for the consumer would make her as happy as she was originally (keep her on the same indifference curve). This is shown on the following diagram showing that a rise from 4 to $11.4 = 2 \bullet 5.7$ would do the trick.



Perfect indexing

I could produce the perfect index because I could use my knowledge of the consumer's preferences. Such knowledge is not generally available and furthermore different consumers have different preferences. A personal price index for me may be a bad index for you!

The **microeconomic theory of index numbers** (Diewert) considers these aspects and more.