Baby Busts and Baby Booms: The Fertility Response to Shocks in Dynastic Models

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Abstract

After the fall in fertility during the demographic transition, many developed countries experienced a baby bust, followed by the baby boom and subsequently a return to low fertility (BBB event). Demographers have linked these large fluctuations in fertility to the series of 'economic shocks' that occurred with similar timing – the Great Depression, World War II (WWII), the economic expansion that followed and then the productivity slow down of the 1970s. This paper is an attempt to formalize a more general link between fluctuations in output and fertility decisions, in simple versions of stochastic growth models with endogenous fertility. First, we develop initial tools to address the effects of 'temporary' shocks to productivity on fertility choices. These tools are based on a production function, where labor is the only input. Second, we analyze calibrated versions of these models. We can then answer several qualitative and quantitative questions: Is there 'catching-up' in fertility after a period of particularly low fertility? Under what conditions is fertility pro- or counter-cyclical? How large are these effects? Qualitatively, results show that there is no 'catching-up' in this model and that under reasonable parameter values fertility is pro-cyclical. Using the U.S. BBB event as a laboratory, we find that the elasticity of fertility to shocks lays between 1 and 2 and, finally, that in these simple models, productivity shocks capture about 70 percent of the pre-WWII baby bust in the U.S.. For the post-WWII baby boom, the predictions of this simple model are small and happen late compared to the data.

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1 Introduction

After the fall in fertility during the demographic transition, almost the entire 20th century appears to constitute, for the United States and for most other developed countries, a period of unusual deviations in fertility around an otherwise declining trend. First, fertility rates saw an acceleration of the decrease during 1920-1940: the baby bust. This bust was followed by a large upswing during 1940-1965: the baby boom. During the period 1965-1985 the total fertility rate returned to a second slump at even lower levels. Demographers often link these large fluctuations in fertility to the series of economic 'shocks' that occurred with similar timing – the Great Depression, WWII and the economic expansion that followed, and finally the productivity slow down of the 1970's. To economists, this line of argument suggests a more general link between fluctuations in output growth and fertility decisions, of which the baby Bust-Boom-Bust event (BBB) is a particularly stark example.

One complementary hypothesis also asserted in the demography literature is that the baby boom was a 'catching up' phenomenon from a period of relatively low fertility during the Great Depression. (See Whelpton (1954), Freedman et al. (1959), Goldberg et al. (1959), and Whelpton et al. (1966)).

These two hypotheses can be combined to yield the following relationship: current fertility is a stable function of productivity levels (or trends) and the current stock of people in the economy. That is, unusual drops in income (relative to trend) cause fertility to fall, other things equal; vice versa, unusual increases in income (relative to trend) cause fertility to rise. If the increase in income follows a period of below trend growth, the boost to fertility may be even larger because the current stock of people is low compared to the long run 'target' level. Roughly speaking, in this view fertility is a function of the current income shock and of the stock of population, with either long run average income or long run 'trend' growth rates determining the target level of population, or of family size. In this paper we formalize this link by combining simple versions of the stochastic growth model and the dynastic model of endogenous fertility a la Barro-Becker. We find that the most simple models do not exhibit dependence on the stock ('catching up'), but that fertility is indeed pro-cyclical in most cases.[1] We then use these models to approximate the elasticity of fertility to productivity shocks. We find that it lies between 1 and 2. After calibrating to U.S. averages, we find that, through the model, deviations in productivity capture about 70 percent of the pre-WWII baby bust in the U.S.., and about 40 percent of the post-WWII baby boom.

Among recent conjectures relating cyclical movements in income and fertility, perhaps the best known one was advanced by Easterlin (2000). He argues that fertility decisions are based on expected lifetime income relative to material aspirations which are formed in childhood. When expected income is high relative to individual aspirations, fertility is high, and vice versa. For the women born in the very early years of the twentieth century who were making fertility decisions in the late 1920s and during the 1930s, expected lifetime income was relatively low due to the Great Depression – hence, the baby bust. Vice versa, since the baby boom mothers grew up in bad times (the 1930s) and therefore had low material aspirations – while they were making fertility decisions during good times (i.e. expected lifetime income was high), they had many children. Operationally, low income for today's generation implies high fertility for the next generation, especially if its expected lifetime income is particularly high.[2]

It is not hard to see that this version of the relationship between (rela-

[1] One difficulty with the 'catching up' hypothesis that needs to be addressed, according to both demographers and economists, is that, in the data, this 'catching up' clearly does not take place for a given woman (see Greenwood et al. (2005) and section 3.1 below for documentation). That is, completed fertility was low for both the women immediately preceding and immediately following the baby boom mothers. So, if there is 'catching up', it is in a dynastic (aggregate) sense, not at the individual level. That is, in a dynastic model, it is quite possible that the dynasty purposefully 'catches up,' although this need not be observed for any particular generation of women. This is a distinction that we briefly discuss as well.

[2] Formally, one version of this story would be to assume that there is habit formation in consumption, as in much of the recent literature on asset pricing.

tive) income and fertility decisions, as well as the previous one linking above (below) average growth and high (low) fertility are, in fact, 'dynamic variations' of the Malthusian hypothesis, both in spirit and in their substantive predictions (see Malthus (1798)). Recall that, in the traditional Malthusian view, the long run population level is determined by a fixed natural ratio between available economic resources and population size. When income per capita - i.e. labor productivity - increases above this natural level, fertility also increases until the long run ratio is reestablished, and vice versa for periods of economic crisis. The prediction that periods of unusually high mortality, during which population is depleted while economic resources remain unchanged, should be followed by years of above average fertility, and vice versa is not far from the one considered here: periods of very harsh economic conditions, in which per capita income decreases below the natural level, are also periods in which fertility decreases. Cipolla (1962), Simon (1977), and Boserup (1981) are some of the best historical renditions of such a 'generalized' Malthusian view, and to them we refer for the many details we must by force omit in our brief historical overview below.

While one can see from the above that the theory being considered here dates back to the very origins of modern economic demography, surprisingly little has been done to formally address the link between productivity shocks and fertility in a stochastic model of optimal fertility choice. This paper aims at filling this gap by investigating the theoretical and quantitative implications of this link in a stochastic version of the traditional dynastic model of endogenous fertility (see Becker and Barro (1988) and Barro and Becker (1989)). Formally, in these models the size of population in period t, N_t, plays a role much like the capital stock in a standard stochastic growth model. Analogously, fertility plays the role of investment. This analogy is sometimes imperfect since, for example, in the Barro-Becker rendition of the dynastic growth model, N_t also enters the utility function of the planner. Thus, in truth N_t has features that are a mixture of capital and consumption in the stochastic growth model. Also, if children cost time, there is an opportunity cost component through wages – absent in the standard stochastic growth model with capital – that makes investment in children cheap in bad times and expensive in good times.[3]

Given those provisos, recall the simple intuition from the single sector growth model with productivity shocks. There is a fundamental desire to smooth consumption due to the concavity of the utility function. Because of this, in a period when productivity is lower than average (and, as a result, output is correspondingly low), agents lower investment to smooth consumption. When the shock is high, the opposite occurs. Thus, the growth rate of K is high when the shock is high and low when it is low. In the case where the analogy to N_t in the endogenous fertility models holds, this implies that the growth rate of N_t, i.e., the fertility rate, is high when the shock is high, and low when the shock is low. These first order deviations induced by variations in current productivity, can be either damped or magnified by the particular type of production function one adopts.

In this paper, we take the simplest case by considering a stochastic version of a Barro-Becker type model where we abstract from all other inputs besides pure labor (such as physical and human capital or land). First, we derive homogeneity properties of the model and find that, due to these, fertility depends on the current productivity shock only, and not on the size of the current stock, N_t. This implies that, while there is a link between productivity shocks and fertility, this type of model does not exhibit 'catching up' fertility due to low fertility in the past.

Next, we analyze a version of the model in which population does not en-

[3]Although it seems unlikely that fertility decisions are affected by quarter to quarter fluctuations in productivity (as addressed in the business cycle literature), longer fluctuations, such as the Great Depression where productivity was below trend for about 10 years, the post-war period where it was above trend for about the same number of years, and the extended productivity slow-down in the seventies, are likely to affect parents outlook on their children's future well being, and therefore their current fertility decisions. In the present context, temporary shocks should therefore be understood as extended swings around a very long term trend.

ter the dynastic planner's utility function. This requires a particular configuration of parameters in the Barro-Becker type preferences. In this case only total consumption by all members of the dynasty enters the utility function. This specification simplifies the model and reduces it to one which is analogous to a stochastic Ak model (see Jones and Manuelli (1990) and Rebelo (1991)). Under the additional assumption that the shocks to productivity are $i.i.d.$, we give analytical characterizations of the model for particular cases and show that, in most of them, fertility rates are pro-cyclical.

In Section 3, present data on the U.S. bust-boom-bust event and perform quantitative experiments on this version of the model. To do this, we first calibrate the model parameters to various averages in the data. We then use actual magnitudes of productivity shocks from 1910 to 1970 and compare the predictions of the model in terms of deviations in fertility rates to the data. We consider two alternative specifications of the model: (1) all costs of raising children are goods costs; (2) all costs are time costs. For (1), we look at two examples. In the first, survival is zero. That is, the dynasty has to build a new stock of people every period. In this case, the elasticity of fertility to the current shock is 1. Second, we consider a case in which survival of working age people over a 10 year period is 80 percent. In this case, the elasticity of fertility to productivity shocks increases to 2. The example with full depreciation of the population is interesting because it corresponds most closely to that of the original Barro-Becker model. When survival is 80 percent the expected working life is 50 years which is more realistic. In the case where all costs are in terms of time, we only consider the 80 percent survival rate and find that the elasticity is 1.9.

Since during the Great Depression productivity was about 10 percent below trend, we get about one-third to two-thirds of the 26 percent downward deviation in fertility in the first case and about two-thirds in the second. For the baby boom, the model does less well. There was a take-off in productivity after WWII, but it was more pronounced in the 1960s than in the 1950s.

Thus, the model predictions are smaller and happen later than what is seen in the data, capturing about 25 to 40 percent of the baby boom itself.[4]

2 A Simple Model of the Fertility Response to Productivity Shocks

In this section, we lay out a model of the response of fertility to period by period stochastic movements in Total Factor Productivity (TFP). To do this, we use a model of fertility based on that developed in Becker and Barro (1988) and Barro and Becker (1989) (Barro-Becker henceforth). The simplification that we make is to assume that there is no physical (or human) capital in the model. Thus, the flow of income is solely due to wage income. On the other hand, we add a stochastic component to the basic Barro-Becker model.

First, we derive homogeneity properties of the model and find that, due to these, fertility depends on the current productivity shock only, and not on the size of the current stock, N_t. This implies that, while there is a link between productivity shocks and fertility, this type of model does not exhibit 'catching up' fertility due to low fertility in the past.[5] Second, we show that by reinterpreting the discount factor, this model is equivalent to one in which productivity follows an exogenous exponential growth process. As long as costs of children grow at that same rate, the population growth rate is stable, while consumption per capita grows at the exogenous rate.

Next, we analyze a version of the model in which population does not enter the dynastic planner's utility function. This requires a particular configuration of parameters in the Barro-Becker type preferences. In this case

[4] All of these statements are contingent on a particular way of identifying the 'shocks' to productivity in the TFP time series. And the results will certainly be affected by how this issue is treated. We are currently studying alternatives along this line.

[5] However, in extensions of the model that involve changing the state space (to include physical capital or a 'vintage structure' to the lifetimes of the dynasty), the phenomenon may very well occur. See Boldrin et al. (2006a).

only total consumption by all members of the dynasty enters the utility function. This specification simplifies the model and reduces it to one which is analogous to a stochastic Ak model.[6] Under the additional assumption that the shocks to productivity are *i.i.d.*, we give analytical characterizations of the model for two particular cases: first, the case in which all child care costs are goods costs; second, the case in which children only cost time.

In the first case, we show analytically that fertility is, indeed, pro-cyclical – a high shock relative to trend is associated with higher than average fertility. In fact, it is a linear function of the shock, with the sign of the slope independent of preference parameters. Thus, in this case the link between productivity and fertility is qualitatively consistent with the BBB episode. The magnitude of the effect, however, depends on the length of the period which affects the depreciation or, more appropriately, the mortality rate in the economy. In this setting, the magnitude of the effect on fertility is decreasing in the survival rate.

When all child care costs are in terms of time, matters become more complicated. Since shocks to productivity affect wages, they also affect the cost of children. In this case, we again give analytical results for the case of *i.i.d.* shocks. We find that whether fertility is pro- or counter-cyclical depends on the parameters of preferences. With low curvature as is typically assumed in Barro-Becker models, fertility is counter-cyclical, but at the limiting case of log utility it is acyclic. When the curvature of utility with respect to per capita consumption is high, fertility is pro-cyclical.[7]

[6]Even though dynasty size does not enter the planner's utility function in this case, N_t is chosen to be positive in all periods because it is essential in production.

[7]The central role played by the curvature of utility to determine if fertility is pro- or counter-cyclical suggests that one should also consider models of endogenous fertility in which the latter is not driven by the dynastic utility motive, but by different ones, such as the 'late age insurance' motive, formalized in Boldrin and Jones (2002). In such context, we conjecture, fertility is always pro-cyclical, both when child care costs are in terms of goods and of time.

2.1 Model setup

Households are alive for one period as a child, where no decisions are made. In subsequent periods they are adults and make consumption and fertility choices. With probability π, adults survive to the next period.[8] Adults care about consumption, the number of children and their children's future utility. In every period they decide how many children to have and how much to consume given their income. All income is stochastic labor income. Children cost θ which may be a goods cost or a time cost.

We can formulate the dynastic head planner's problem as follows. [9]The dynasty head at time zero solves the maximization problem:

$$Max_{\{C_t,N_t\}} \qquad E_0\left(\sum_{t=0}^{\infty}\beta^t g(N_t(s^{t-1}))u\left[\frac{C_t(s^t)}{N_t(s^{t-1})}\right]\right)$$

subject to:

$$C_t(s^t)+\theta_t(s^t)N_{bt}(s^t)\leq N_t(s^{t-1})w_t(s^t),$$

$$N_{t+1}(s^t)\leq \pi N_t(s^{t-1})+N_{bt}(s^t),$$

N_0 fixed.

Where $s^t=(s_0,s_1,....,s_t)$ is the history of shocks up to and including period t, w_t is the stochastic process for wages (assumed to be a function of the shocks), θ_t is the cost of raising a child born in period t, C_t is aggregate consumption for the dynasty in the period (assumed split across all individuals of working age), N_{bt} is the number of new children born in the dynasty in period t and N_t is the number of dynasty members of working age alive in the period.

[8]Note that π is independent of 'age'. This is a short-cut to an explicit age-structure that keeps the state space small but allows longer than one period life-times. In the Appendix, we briefly address formulations of the model that include an explicit age-structure and age-specific mortality rates.

[9]Alvarez (1999) shows the equivalence between the equilibrium allocations from a sequence of individual problems and the dynastic head's problem in this type of model.

Note that each individual alive in period t is assumed to have a survival probability to period $t+1$ of π. This is unusual for a Barro-Becker fertility model, where it is usually assumed that individuals only have one period of active decision making. This corresponds to the assumption that $\pi = 0$, one of the special cases we will discuss below. However, since some of the TFP movements that we want to discuss are at frequencies higher than a generation (in the fertility sense), it will be convenient to also consider cases in which $\pi > 0$. Because of this choice of functional form (admittedly a gross simplification of actual dynastic survival processes), note that the model is equivalent to one in which the stock, N, depreciates at rate $\delta = 1 - \pi$ over each period. This will allow us to use the analogy to a stochastic Ak model with less than full depreciation below. We have assumed that the flow utility function is of the form $U(C, N) = g(N)u(\frac{C}{N})$, i.e., utility depends on both the size of the dynasty and per capita consumption. Assuming that $g(N) = N^{\eta}$ and $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, we can rewrite this problem as:

$$\text{P1} \qquad Max_{\{C_t, N_t\}} \qquad \sum_{t=0}^{\infty} \beta^t N_t^{\eta} \left[\left[\frac{C_t}{N_t} \right]^{1-\sigma} /(1-\sigma) \right] = \sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to:

$$C_t + \theta_t \left(N_{t+1} - \pi N_t \right) \leq N_t w_t.$$

There are two sets of parameter restrictions that satisfy the natural monotonicity and concavity restrictions for this functional form, both in terms of the aggregate, or dynasty variables, (C, N), and in terms of per capita values, $(N, c) = (N, \frac{C}{N})$. These are: i) $0 < \eta < 1$, $0 < \sigma < 1$ and $0 \leq \eta + \sigma - 1 < 1$, and ii) $\sigma > 1$ and $\eta + \sigma - 1 \leq 0$. We will explore both of these options below.

Notice that if $\eta + \sigma - 1 = 0$, then N does not enter the period utility function except in aggregate consumption, and hence, N plays exactly the same role in this model as K does in a stochastic Ak model.[10] There is one

[10]See Jones and Manuelli (1990) and Rebelo (1991), seminal papers on this model.

twist however. That is θ_t is also stochastic, at least in the case where child-rearing is modelled as a time cost. In that case, $\theta_t = bw_t$ and hence, if w_t is stochastic, so are child-rearing costs. That is, the periods when productivity is high are also those when capital is expensive. Also, in this case aggregate consumption, C, grows at the same rate as N, but per capita consumption is constant (without shocks). Other than that, the analogy is very close.

Let us introduce one additional piece of notation. Let N_{ft} be the fertile part of the working population, i.e. $N_{ft} = \lambda_t N_t$. Then children per fertile person - the fertility rate, n_{bt} can be expressed as $n_{bt} = \frac{N_{bt}}{N_{ft}}$. This is the model quantity that we will identify with the Total Fertility Rate (TFR) in the Data.

2.2 Homogeneity properties of the planner's problem

Recall that we want to study solutions to maximization problems of the form:

$$P(N_0, s_0) \qquad Max_{\{C_t, N_t\}} \qquad U_0(\{C_t, N_t\}) = E_0\left[\sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} \frac{C_t^{1-\sigma}}{1-\sigma}\right]$$

subject to:

$$C_t + \theta_t N_{t+1} \leq N_t \left(w_t + \theta_t \pi\right),$$

N_0 given.

Let $V(N_0, s_0)$ denote the maximized value of the objective at a solution (assuming one exists) and let $\{(C_t^*(N_0, s_0), N_t^*(N_0, s_0)\}_{t=0}^{\infty}$, denote the solution itself.

Proposition 1. *$V(\lambda N_0, s_0) = \lambda^{\eta} V(N_0, s_0)$ and*
$\{(C_t^*(\lambda N_0, s_0), N_t^*(\lambda N_0, s_0)\}_{t=0}^{\infty} = \lambda\{(C_t^*(N_0, s_0), N_t^*(N_0, s_0)\}_{t=0}^{\infty}$.

Proof.
Step 1: $\{(C_t, N_t\}_{t=0}^{\infty}$ is feasible for $P(N_0, s_0)$ if and only if $\{(\lambda C_t, \lambda N_t\}_{t=0}^{\infty}$ is feasible for $P(\lambda N_0, s_0)$.

Step 2: $U_0(\lambda\{C_t, N_t\}) = \lambda^\eta U_0(\{C_t, N_t\})$.
Step 3: If the claim is false, then there is something better for $P(\lambda N_0, s_0)$ than $\lambda\{(C_t^*(N_0, s_0), N_t^*(N_0, s_0)\}_{t=0}^{\infty}$. Call this feasible plan $\{(C_t', N_t')\}_{t=0}^{\infty}$. From Step 1, the plan $\frac{1}{\lambda}\{(C_t', N_t')\}_{t=0}^{\infty}$ is feasible for $P(N_0, s_0)$. From Step 2, $\frac{1}{\lambda}\{(C_t', N_t')\}_{t=0}^{\infty}$, gives higher utility than $\{(C_t^*(N_0, s_0), N_t^*(N_0, s_0)\}_{t=0}^{\infty}$, a contradiction. □

This is a standard homogeneous/homothetic type argument. Because of this result, and because the problem is stationary if $(w(s), \theta(s))$ is assumed to be a first order Markov process, we can characterize the solution through Bellman's Equation:

$$(BE) \qquad V(N, s) \equiv \sup_{(C,N')} N^{\eta+\sigma-1}C^{1-\sigma}/(1-\sigma) + \beta E\left[V(N', s')|s\right]$$

$$\text{s.t.} \qquad C + \theta(s)N' \leq \left(w(s) + \theta(s)\pi\right) N.$$

Because of Proposition 1, this can be rewritten as:

$$(BE) \qquad V(N, s) \equiv \sup_{(C,N')} N^{\eta+\sigma-1}C^{1-\sigma}/(1-\sigma) + \beta E\left[(N')^{\eta} V(1, s')|s\right]$$

$$\text{s.t.} \qquad C + \theta(s)N' \leq \left(w(s) + \theta(s)\pi\right) N,$$

or,

$$(BE) \qquad V(N, s) \equiv \sup_{(C,N')} N^{\eta+\sigma-1}C^{1-\sigma}/(1-\sigma) + \beta \left(N'\right)^{\eta} E\left[V(1, s')|s\right]$$

$$\text{s.t.} \qquad C + \theta(s)N' \leq \left(w(s) + \theta(s)\pi\right) N.$$

Where the last step follows since N' is a function of s alone (and not s').

Now, let $D(s) \equiv E\left[V(1, s')|s\right]$ to obtain:

$$(BE) \qquad V(N, s) \equiv \sup_{(C,N')} N^{\eta+\sigma-1}C^{1-\sigma}/(1-\sigma) + \beta \left(N'\right)^{\eta} D(s)$$

$$\text{s.t.} \qquad C + \theta(s)N' \leq \left(w(s) + \theta(s)\pi\right) N.$$

From this, it follows that the FOC's for the problem are:

$$(FOC1) \qquad \frac{N^{\eta+\sigma-1}C^{-\sigma}}{1} = \frac{\beta\eta(N')^{\eta-1}D(s)}{\theta(s)},$$

$$(FOC2) \qquad C + \theta(s)N' = (w(s) + \theta(s)\pi)\, N.$$

These equations can be simplified to some extent. We have:

$$(FOC1) \qquad \left[\frac{C}{N}\right]^{-\sigma} = \frac{\beta\eta D(s)}{\theta(s)} \left[\frac{N'}{N}\right]^{\eta-1},$$

so that,

$$(FOC1) \qquad n = \frac{N'}{N} = \left[\frac{C}{N}\right]^{\frac{\sigma}{1-\eta}} \left[\frac{\theta(s)}{\beta\eta D(s)}\right]^{1/(\eta-1)}.$$

Substituting into $FOC2$ we get:

$$(FOC2) \qquad C + \theta(s)N' = (w(s) + \theta(s)\pi)\, N,$$

$$(FOC2) \qquad \frac{C}{N} + \theta(s)\frac{N'}{N} = (w(s) + \theta(s)\pi)\,,$$

$$(FOC2) \qquad \frac{C}{N} + \theta(s) \left[\frac{C}{N}\right]^{\frac{\sigma}{1-\eta}} \left[\frac{\theta(s)}{\beta\eta D(s)}\right]^{1/(\eta-1)} = (w(s) + \theta(s)\pi)\,.$$

It follows that $\frac{C}{N}$ and $\frac{N'}{N}$ are functions of the current shock only (although C and N' are not) and NOT the current level of the stock, N. Since this property plays a role in the ability of this type of model to exhibit a 'catching up' of fertility after a low shock, we state this as a formal proposition:

Proposition 2. *The solution to the Planner's Problem is to find* $\frac{C}{N}$ *and* $\frac{N'}{N}$ *which solve:*

$$\frac{C}{N} + \theta(s) \left[\frac{C}{N}\right]^{\frac{\sigma}{1-\eta}} \left[\frac{\theta(s)}{\beta\eta D(s)}\right]^{1/(\eta-1)} = (w(s) + \theta(s)\pi)$$

and,

$$\frac{N'}{N} = \left[\frac{C}{N}\right]^{\frac{\sigma}{1-\eta}} \left[\frac{\theta(s)}{\beta\eta D(s)}\right]^{1/(\eta-1)}.$$

Thus, the growth rate in population, $\frac{N'}{N}$ *is a function of the current shock only, and not the size of the current stock,* N*.*

Furthermore, if the fraction of fertile people in the population, λ_t, is independent of time, i.e. $\lambda_t = \lambda$, then the fertility rate also only depends on the current shock, s_t, and not the current stock, N_t

Proposition 3. *Let $N_f = \lambda N$. If λ is independent of time, then* $n_b = \frac{N_b}{N_f} = \frac{N' - \pi N}{N_f} = \frac{1}{\lambda}\left(\frac{N'}{N} - \pi\right).$

That is, the fertility rate is independent of the size of the current stock, but, through $\frac{N'}{N}$, depends on this period's productivity shock.

This is an important qualitative property of the model. That is, the central idea behind the notion that the baby boom is 'catching up' is that the size of the dynasty is 'too small' (relative to trend) at the end of WWII because of the baby bust. Thus, fertility is increased so as to bring the size of the 'stock' up to its desired level. However, due to this result, it follows that fertility in the model does not depend on the size of the dynasty, but only on the shock. Thus, this kind of model can never exhibit 'catch up' fertility of this type. As we discuss in Boldrin et al. (2006a), where we enlarge the state space to include either physical capital or age vintages of people, we find that such stock dependence occurs in certain cases.

2.3 An aside on trend growth in productivity

In this section we add to the previous analysis trend growth in productivity. This is assumed to be exogenous.

Thus, we will study solutions to maximization problems of the form:

$$P(\gamma, \beta; N_0, s_0) \qquad Max_{\{C_t, N_t\}} \qquad U_0(\{C_t, N_t\}) = E_0\left[\sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} \frac{C_t^{1-\sigma}}{1-\sigma}\right]$$

subject to:

$$C_t + \gamma^t \theta_t N_{t+1} \leq N_t \gamma^t \left(w_t + \theta_t \pi\right),$$

N_0 given.

We assume that $\gamma \geq 1$ is an exogenous constant.

Note that a sequence $\{C_t, N_t\}$ is feasible for this problem if and only if it satisfies:

$$\frac{C_t}{\gamma^t} + \theta_t N_{t+1} \leq N_t \left(w_t + \theta_t \pi \right)$$

Thus, defining $\hat{C}_t = \frac{C_t}{\gamma^t}$, this Problem can be written equivalently as:

$$P(\gamma, \beta; N_0, s_0) \qquad Max_{\{\hat{C}_t, N_t\}} \qquad U_0(\{\hat{C}_t, N_t\}) = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\gamma^{1-\sigma} \right)^t N_t^{\eta+\sigma-1} \frac{\hat{C}_t^{1-\sigma}}{1-\sigma} \right]$$

subject to:

$$\hat{C}_t + \theta_t N_{t+1} \leq N_t \left(w_t + \theta_t \pi \right),$$

N_0 given.

Note that this problem is the same as:

$$P(1, \hat{\beta}; N_0, s_0) \qquad Max_{\{\hat{C}_t, N_t\}} \qquad U_0(\{\hat{C}_t, N_t\}) = E_0 \left[\sum_{t=0}^{\infty} \hat{\beta}^t N_t^{\eta+\sigma-1} \frac{\hat{C}_t^{1-\sigma}}{1-\sigma} \right]$$

subject to:

$$\hat{C}_t + \theta_t N_{t+1} \leq N_t \left(w_t + \theta_t \pi \right),$$

N_0 given.

where $\hat{\beta} = \beta \gamma^{1-\sigma}$.

Note that this problem has no exogenous growth in it. Thus, we have:

Claim 1. *If $\{(\hat{C}_t^*, N_t^*)\}$ solves the problem $P(1, \hat{\beta}; N_0, s_0)$ for some $(1, \hat{\beta}; N_0, s_0)$, then $\{(\gamma^t \hat{C}_t^*, N_t^*)\}$ is the solution to the problem $P(\gamma, \hat{\beta}/\gamma^{1-\sigma}; N_0, s_0)$.*

Thus, to solve the problem with exogenously growing TFP, solve the one with $\gamma = 1$ and $\hat{\beta} = \beta\gamma^{1-\sigma}$, and then multiply the C sequence by γ^t. It follows that if $\frac{C}{N}$ is constant (or converges to a constant) in the solution to the problem with no growth, then, $\frac{C}{N}$ grows at rate γ in the one with exogenous growth.

Because of this result, we will abstract from trend growth through most of the remainder of the paper. In those cases where the solution to the model depends on the discount factor, we will use this result to calibrate to the appropriate discount factor in the detrended model.

2.4 The stochastic Ak analogy

In this section, we will specialize the model outlined above even further by assuming that $\eta + \sigma = 1$ and that the $\{s_t\}$ are i.i.d. There are several simplifications that occur under these assumptions. These are:

1. As noted above, in this case, the value function is homogeneous of degree $1 - \sigma$ (since $\eta = 1 - \sigma$) in N_0, $V(\lambda N_0, s_0) = \lambda^{1-\sigma} V(N_0, s_0)$.

2. Define $D(s) \equiv E[V(1, s')|s]$ for what comes below to simplify notation. Since the shocks are *i.i.d.*, it follows that $D(s) = E[V(1, s')|s] = E[V(1, s')] = D$.

3. $(FOC1)$ from Bellman's Equation (given above) simplifies to:

$$(FOC1) \qquad N' = \left[\frac{\beta\eta D}{\theta(s)}\right]^{1/\sigma} C.$$

Furthermore, throughout this section, we will assume that:

$$w(s) = A(s) = As,$$

where the s are i.i.d. with $E(s) = 1$.

Finally, we will consider two extreme cases for the form of $\theta(s)$. In the first, we assume that only goods are needed to raise children, with $\theta(s) \equiv a$. In the second we assume that only time is used, $\theta(s) = bAs$.

2.4.1 Goods cost only ($\theta_t = a$)

Here we assume that all costs of raising children can be summarized as a time invariant cost stated in terms of the consumption good. In this case an analytic solution to the Planner's Problem can be given. It is summarized in:

Proposition 4. *Suppose $\theta_t = a$ and assume that the shocks are i.i.d.. Then the problem has an analytical solution given by:*

$$\begin{cases} C = \varphi\left(As + a\pi\right)N \\ N' = \frac{(1-\varphi)}{a}\left(As + a\pi\right)N \end{cases}$$

where: $\varphi = \frac{1}{1+a^{\frac{\sigma-1}{\sigma}}\left(E(V(1,s'))\beta(1-\sigma)\right)^{\frac{1}{\sigma}}}$.

Both, the fertility rate and per capita consumption are pro-cyclical.

Proof. See Appendix. □

Note that φ and B depend on A, a and $E\left[(As + a\pi)^{1-\sigma}\right]$. Using the characterization in Proposition 4, it follows that fertility today (children per fertile person) is given by:

$$n_{bt} = \frac{N_{bt}}{\lambda_t N_t} = \frac{(1-\varphi)(As_t + a\pi)}{\lambda_t a} - \frac{\pi}{\lambda_t} = \frac{(1-\varphi)A}{\lambda_t a} s_t - \frac{\varphi\pi}{\lambda_t}, \text{ and,}$$

$$c_t = \frac{C_t}{N_t} = \varphi\left(As_t + a\pi\right) = \varphi A s_t + \varphi a \pi.$$

These depend on today's shock only. Thus, both the fertility rate and per capita consumption follow TFP movements procyclically in this case.

2.4.2 Time cost only ($\theta_t = bA(s_t)$)

In this section, we switch to the other extreme, and assume that all child-rearing costs are in terms of time for the parents. That is, $\theta_t = bA(s_t) = bAs_t$. Under this assumption, the Planner's Problem has an analytic solution as summarized in the following proposition.

Proposition 5. *Assume that $\theta_t = bA(s_t) = bAs_t$ and that the shocks are i.i.d.. Then the problem has an analytical solution given by:*

$$\begin{cases} C = \varphi(s)\left(1 + b\pi\right) A(s) N \\ N' = \frac{(1-\varphi(s))}{b}\left(1 + b\pi\right) N \end{cases}$$

where: $\varphi(s) = \frac{1}{\left[1+(bAs)^{1-1/\sigma}[\beta(1-\sigma)E[V(1,s')]]^{1/\sigma}\right]}$.

It follows that $\frac{N'}{N}$ is increasing in s if $\sigma > 1$ and decreasing in s if $\sigma < 1$. Moreover $\frac{C}{N}$ is increasing in s if

- *$\sigma < 1$ or*
- *$\sigma > 1$ and $s > 1 - 1/\sigma$*

and decreasing in s otherwise.

Proof. See Appendix. □

Notice that if $\sigma < 1$, then $E\left[V(1, s')\right] > 0$ and $1-1/\sigma < 0$ and so $s^{1-1/\sigma}$ is decreasing in s. It follows that in this case, $\varphi(s)$ is increasing in s. In this case then, it follows immediately that $\frac{N'(N,s)}{N} = \frac{1-\varphi(s)}{b}\left(1 + b\pi\right)$ is decreasing in s, i.e. fertility is counter-cyclical, while $\frac{C(N,s)}{N} = \varphi(s)\left(1 + b\pi\right) A(s)$ is increasing in s, i.e. per capita consumption is pro-cyclical.

In the opposite case, if $\sigma > 1$, then $E\left[V(1, s')\right] < 0$ and $1 - 1/\sigma > 0$ and so $s^{1-1/\sigma}$ is increasing in s. It follows that in this case, $\varphi(s)$ is decreasing in s and hence N'/N is increasing in s. Taking derivatives, one can show that C/N is increasing in s if $s > 1 - 1/\sigma$ and decreasing in s otherwise.

Hence, in the case where all costs of children are in terms of time, fertility and consumption move in opposite directions, except in the case where σ is relatively high (consumption smoothing is a priority) while the shock is particularly low. This is the only instance where consumption is not procyclical.

The following characterization of the value function will be used in our numerical exercise. Using Proposition 5, we can characterize the Value Function by:

$$(BE) \qquad V(N,s) \equiv [C(N,s)]^{1-\sigma}/(1-\sigma) + \beta\left(N'(N,s)\right)^{1-\sigma} D$$

$$= [\varphi(s)\,(1+b\pi)\,AsN]^{1-\sigma}/(1-\sigma) + \beta\left[\tfrac{(1-\varphi(s))(1+b\pi)AsN}{bAs}\right]^{1-\sigma} D = B(s)\,[sN]^{1-\sigma},$$

where

$$B(s) = \left[\tfrac{(\varphi(s)(1+b\pi)A)^{1-\sigma}}{1-\sigma} + \beta\left[\tfrac{(1-\varphi(s))(1+b\pi)A}{bAs}\right]^{1-\sigma} D\right]$$

$$= (1+b\pi)^{1-\sigma}\left[\tfrac{(\varphi(s)A)^{1-\sigma}}{1-\sigma} + \beta\left[\tfrac{(1-\varphi(s))}{bs}\right]^{1-\sigma} E\left[V(1,s')\right]\right].$$

Thus, integrating, we obtain:

$$E\left[V(1,s)\right] = E\left[B(s)1 \times s^{1-\sigma}\right] = E\left[B(s)s^{1-\sigma}\right].$$

In the next section, we use the above characterization to compute value and policy functions to simulate the U.S. fertility experience during the 20th century.

3 The U.S. Experience, 1900-2000

In this section, we first lay out the basic facts about the time paths of TFP and fertility in the U.S. over the 20th century. We find that the correlation and timing of events are, qualitatively, very much in line with the predictions of the model above.

Next we perform quantitative experiments on the version of the model analogous to the stochastic Ak model (see Section 2.4). To do this, we calibrate to first moments of our data. We then use actual magnitudes of productivity deviations (shocks) for every decade from the 1910s to the 1960s and compare the predictions of the model in terms of percent deviations in fertility rates to the data. We consider two alternative specifications of the model: (1) all costs of raising children are goods costs; (2) all costs are time costs. For (1), we look at two examples. In the first, survival is zero. That is, the dynasty has to build a new stock of people every period. In this case, the elasticity of fertility to the current shock is 1. Second, we consider a case in which survival of working age people over a 10 year period is 80 percent. In this case, the elasticity of fertility to productivity shocks increases to 2. The example with full depreciation of the population is interesting because it corresponds most closely to that of the original Barro-Becker model. When survival is 80 percent the expected working life is 50 years which is more realistic. In the case where all costs are in terms of time, we only consider the 80 percent survival rate and find that the elasticity is 1.9.

Since during the Great Depression productivity was about 10 percent below trend, we get about one-third to two-thirds of the 26 percent downward deviation in fertility in the first case and about two-thirds in the second. For the baby boom, the model does less well. There was a take-off in productivity after WWII, but it was more pronounced in the 1960s than in the 1950s. Thus, the model predictions are smaller and happen later than what is seen in the data, capturing about 25 to 40 percent of the baby boom itself. All in all however, given the simplicity of the model, we view this as quite a success.

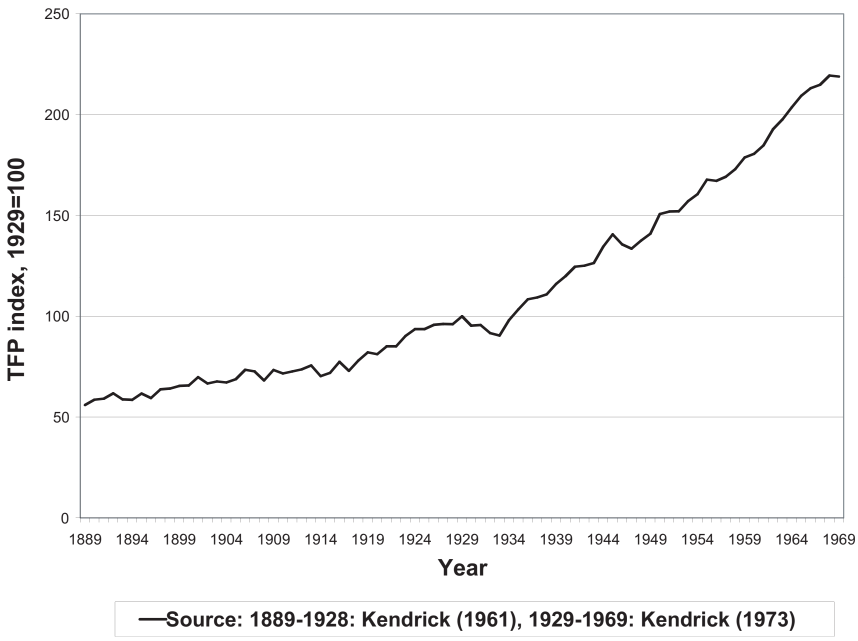
3.1 Data

In this section, we lay out the basic facts about the time paths of TFP and fertility in the U.S. over the 20th century.[11] We begin with the facts

[11]In Boldrin et al. (2006a) and Boldrin et al. (2006b), we lay out the BBB episode for other developed countries and briefly recall a few among the

pertaining to the growth in productivity as laid out in Kendrick (1961) and Kendrick (1973). As most economists know, this period is one of more or less continued growth in productivity with a few significant breaks. The most significant of these is the Great Depression. Figure 1 shows Kendrick's compilation of Total Factor Productivity (TFP) for the U.S. over the period from 1889 to 1969.

Figure 1: Total Factor Productivity, 1889-1969



The obvious facts about TFP over this period are:

1. The continual upward trend,

2. The marked decline below trend that took place in the 1920s and 1930s,

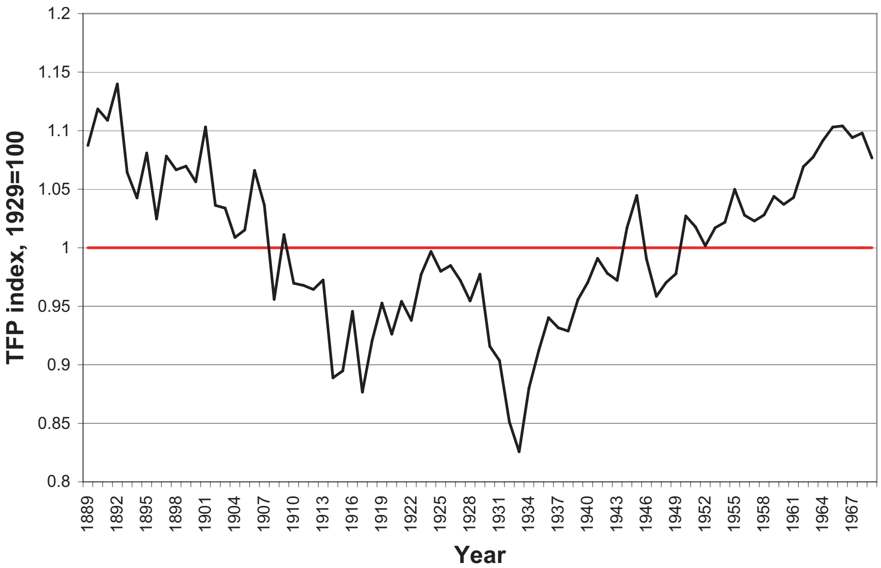
3. The return to trend following WWII.

abundant historical episodes suggesting the existence of a relatively stable relationship between oscillations in income and oscillations in fertility.

The exact sizes of these features of the data depend on how one treats the trend growth in productivity over the period. For example: Was there a common, exogenous growth rate in TFP over the entire period with higher frequency fluctuations (albeit highly autocorrelated fluctuations) around this trend? Or, were there two regimes of growth, one prior to WWII and one after, with higher growth in the second regime? These questions will have an impact on the analysis we present below and because of this, we will try several alternatives. (See more discussion on this in the Appendix.)

After fitting an exponential trend to this series, we obtain the following shocks to productivity over this period. (See Figure 2.)

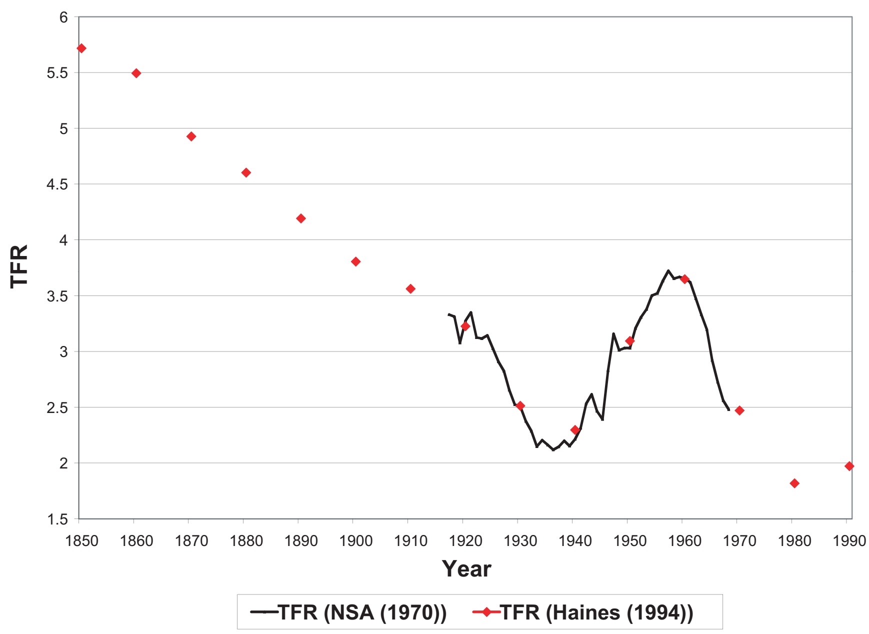
Figure 2: Total Factor Productivity: Deviations from Exponential Trend



As can be seen in the figure, there was a fairly long and deep fall in productivity from trend that took place from 1910 to 1940. In the deepest part of the depression, productivity was about 17 percent below trend. Since that time there was a steady increase up until the late 1960s when productivity was about 10 percent above trend.

This timing of the movements of productivity around trend fits well with the movements in fertility seen in the data. Figure 3 shows the time path of the Total Fertility Rate (TFR) over the period from 1860 to 1968. The figure plots two time series for TFR. The first is the one prepared by Haines (1994) using Census data and hence is available only every 10 years. The second comes from the Natality Statistics Analysis from National Center for Health Statistics. It is available at annual frequencies, but only since 1917. At the beginning of the period, fertility is still in the midst of what is known to demographers as the demographic transition, the marked fall in fertility (and mortality) that has occurred in all developed countries. This fall accelerates from 1920 to 1930, as can be seen in the Haines data. Putting together the NSA and Haines data, it appears that a good description would be:

Figure 3: Total Fertility Rate, 1850-1990



1. High, and fairly constantly decreasing fertility until 1920-21, when it reaches a TFR of about 3.2 children per woman,

2. Acceleration of the rate at which fertility is falling between 1922 and 1932 (from TFR=3.2 to TFR=2.2),

3. Constant, but low, fertility over the period from 1933 to 1940, with the level at about TFR=2.2,

4. Rapidly rising fertility from 1941 to 1956, with TFR going from 2.2 up to 3.6,

5. High, stable fertility from 1957 to 1962 at about TFR=3.6,

6. Falling fertility over the remainder of the period from a TFR of 3.6 in 1963 to about 1.8 around 1980.

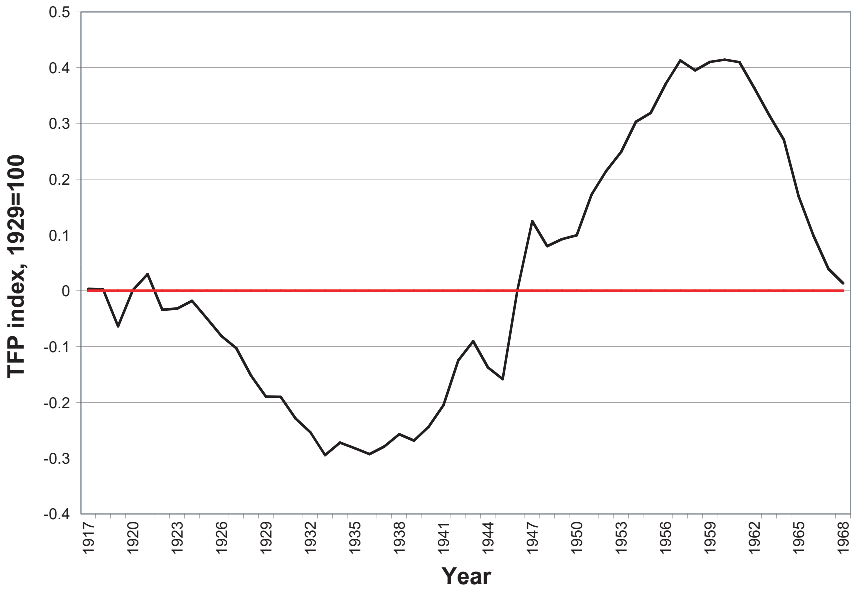
7. Slight recovery in TFR from 1.8 in 1980 to 2.07 in 2000.

Figure 4 shows a similar picture of fertility over this period. It shows the deviations from a fitted, linear trend from 1900 to 1990 from Haines. The deviations at annual frequencies are calculated by subtracting the NSA data for the yearly observations from the fitted trend from Haines.[12] It shows a similar pattern to that described above.

Figure 5 shows the two series of deviations plotted on the same graph. Although it is not perfect, there is an impressive coincidence in timing. The coefficient of correlation between the two annual series for the years 1917 to 1968 is 0.67, which suggests that the U.S. TFR is strongly procyclical during this time period. In sum, TFP was below trend from about 1910 to about

[12]Fitting a trend to the NSA data and calculating deviations from this trend gives virtually identical results. Using the average fertility rate from the NSA data, we find that deviations from the mean are positive until 1926, the baby bust starts later and is slightly smaller while the baby boom is smaller in the 1960s and ends in 1965 - as opposed to 1969 as in Figure 2.

Figure 4: Total Fertility Rate: Deviations from Linear Trend

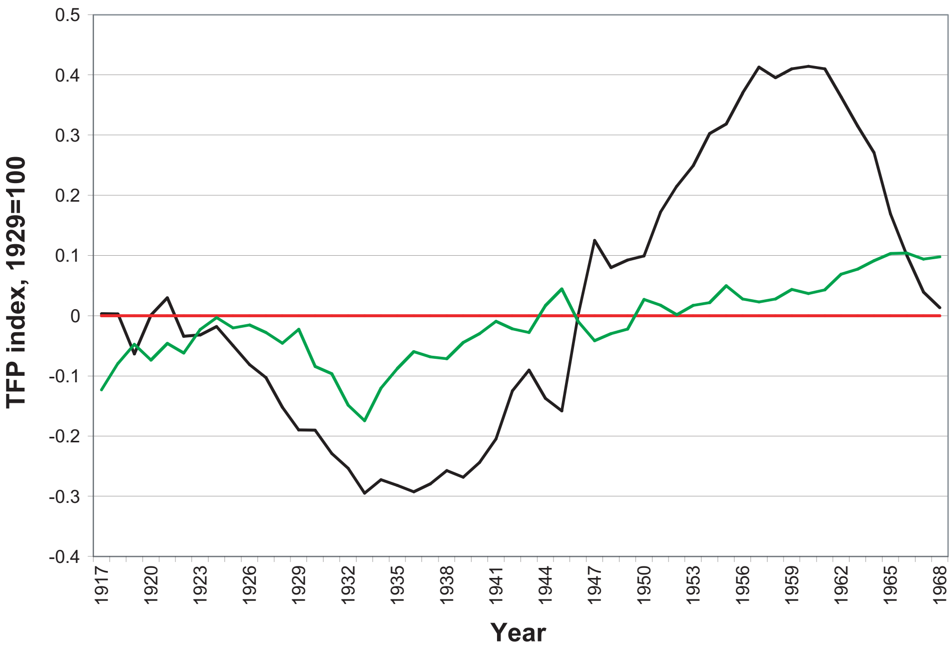


1940, while fertility was below trend from about 1925 to about 1940. They were both above trend over the 1945 to 1965 period with the TFP growth extending beyond this time.

This is not the whole story about the timing of the baby bust and boom, obviously. One possibility is that the baby boom was simply a by-product of delayed fertility of those women whose fertility was low during the period of the baby bust. This is not true, however. In fact, women born roughly between 1905 and 1925 had lower completed fertility than did their mothers or daughters. This can be seen in Figure 6, which shows completed fertility by birth cohort over the period.[13] This reaches a minimum with the 1908 birth cohort then increases back up to a peak with the 1938 birth cohort, falling thereafter. This pattern implies that if the baby boom is a catching up of fertility from the baby bust, it is at the aggregate level, not at the

[13]The data is from Jones and Tertilt (2006), Table A6, column 1.

Figure 5: Total Factor Productivity and Total Fertility Rate: Deviations from Trend

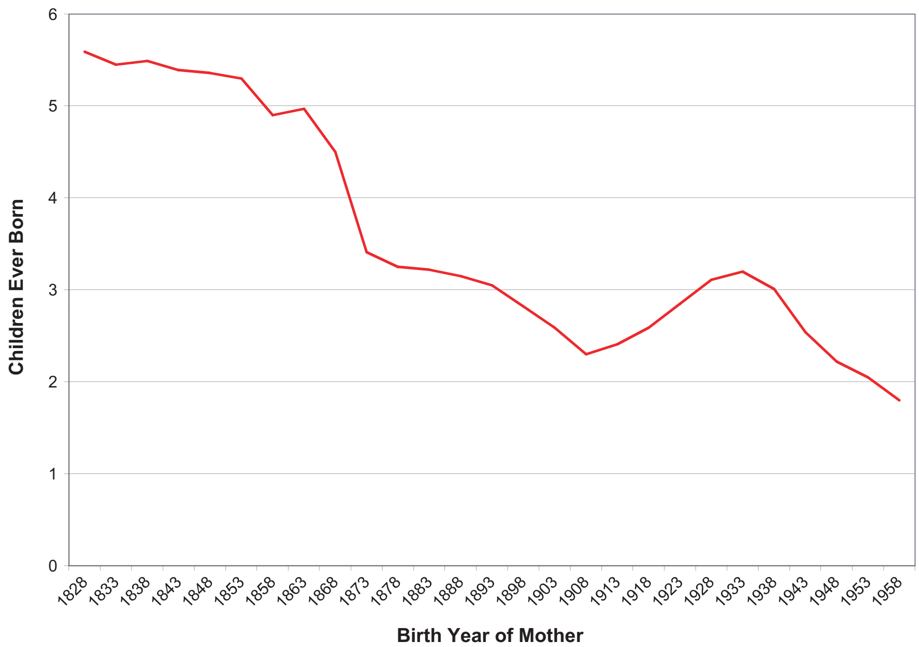


level of the individual woman. See also Greenwood et al. (2005), Doepke and Maoz (2005) and Jones and Tertilt (2006) for more on the make-up, across birth cohorts, of the fertility pattern during the baby boom.

3.2 Simulation

In this section we look at simple calibrated examples of the solution to the model given above. To do this, we must first choose parameter values for the model. A critical choice is the length of the period. Although it seems plausible that parents do consider the likely well-being of their children when making decisions about fertility, it is unlikely that these decisions are driven by quarter to quarter fluctuations of the kind studied in the Real Business Cy-

Figure 6: Children Ever Born, 1815-1960 Birth Cohorts



cle literature. Because of this we assume that a period is 10 years. This also allows us to assume that productivity shocks are *i.i.d.* which is a reasonable approximation for decade to decade movements in TFP. We consider two alternative, simple, specifications of the model: (1) all costs of raising children are goods costs ($\theta(s) = a$); (2) all costs are time costs ($\theta(s) = bw(s) = bAs$). For the first case, we look at two examples: $\pi = 0$ and $\pi = 0.8$. In the second case, we only consider $\pi = 0.8$. The example with full depreciation of the population, $\pi = 0$, is interesting because it corresponds most closely to that of the original Barro-Becker model. When $\pi = 0.8$ the expected working life is 50 years which is more realistic.

In addition to choosing the length of the period and the survival probabilities, we must specify the remaining parameters to fully characterize the decision rules. Throughout we set

- the discount factor, $\beta = 0.97^{10}$ to match an annual interest rate of about 3 percent;
- the growth rate of technological progress, $\gamma = 1.017^{10}$ calculated directly from Kendrick's TFP series (plotted in Figure 1);
- the standard deviation of productivity shocks, $\sigma_s = 0.068$ calculated directly from Kendrick's TFP series (plotted in Figure 1);
- the fraction of the population that is female and of childbearing age, $\lambda = 0.425$ assuming that demographics are stationary and that a generation is 25 years with 85 percent of newborns surviving to age 25, we find that the fraction of fertile people (women) in the population is $\lambda_t = \lambda = 0.5 * 0.85 = 0.425$.

Given these parameters, we then calibrate child costs, $\theta(\cdot)$, and the utility parameter, σ, to match mean fertility and consumption expenditures.

Finally, to compare model responses versus actual time series of fertility deviations, we need the realizations of shocks. From Kendrick's TFP data, the estimated series for s_t decade by decade (beginning with 1910 to 1919) is $(0.935, 0.966, 0.905, 0.987, 1.026, 1.079)$. We discuss alternative treatments of the data and describe some sensitivity to the above parameter values in the Appendix.

3.2.1 Goods cost only ($\theta_t = a$)

Example 1: $\pi = 0$, $T = 10$ years

In this case, it can be seen from Proposition 4 that:

$$n_{bt}(s_t) = \frac{N_{t+1}}{\lambda N_t} = \frac{(1-\varphi)A}{\lambda a} s_t \tag{1}$$

and,

$$c_t(s_t) = \frac{C_t}{N_t} = (\varphi(As_t + a\pi)) = \varphi A s_t \tag{2}$$

.

It follows that $\overline{n_b} = E\left(n_{bt}\left(s_t\right)\right) = \frac{(1-\varphi)A}{\lambda a}$ and thus fertility in the model expressed as percent deviations from the mean are

$$\frac{n_{bt}(s_t)-\overline{n_b}}{\overline{n_b}} = \frac{\frac{(1-\varphi)A}{\lambda a}s_t - \frac{(1-\varphi)A}{\lambda a}}{\frac{(1-\varphi)A}{\lambda a}} = s_t - 1.$$

Therefore, in this case, the elasticity of fertility with respect to the current shock is 1, independent of the assumptions on the parameters. Since the productivity shock in the 1930s was about 10 percent below average, the model predicts that fertility would also be 10 percent below average. Actual fertility was 26 percent below average and hence even this stark version of the model accounts for 40 percent of the baby bust. Similarly, since productivity was 3 and 8 percent above trend in the 1950s and 1960s, respectively, the model generates identical increases in fertility while in the data, fertility was 29 and 23 percent above average.

As we shall see below, the elasticity estimate increases once we relax the full depreciation assumption ($\pi = 0$). In the present case, the entire stock of people must be replenished every period and hence fertility will not fluctuate too much in response to productivity shock for consumption smoothing motives.

Example 2: $\pi = 0.8$, $T = 10$ years

To simulate the effect of changes in s_t on fertility, n_{bt}, we can see from the equations in Proposition 4 that we only need to calibrate φ and A/a.[14] These can be found from data on mean population growth and consumption expenditure shares. In fact,

$$\begin{cases} E(n(s)) = (1-\varphi)\left(A/a + \pi\right) \\ E(c(s))/A = \varphi\left(1 + \frac{\pi}{A/a}\right) \end{cases}$$

[14] In particular we do not need to know the values of σ, β or the distribution parameters of the shock process. Also not needed are A and a separately. Solving for σ, given the values for γ, β and σ_s, and the formula for φ, gives $\sigma = 2.11$.

This is a system of equations in φ and A/a with the following solution.

$$\begin{cases} A/a = \frac{E(n(s))-\pi}{(1-E(c(s))/A)} \\ \varphi = \frac{E(c(s))/A}{(1-E(c(s))/A)} \frac{E(n(s))-\pi}{(A/a+\pi)} \end{cases}$$

We choose to match $E(c(s))/A = 0.75$ because, on average, a family spends about 25 percent of labor income on children and, $E(n(s)) = (TFR * \lambda)^{\frac{10}{25}} = 1.087$ where $n(s) = \frac{N'(N,s)}{N}$, $TFR = 2.9$ is the average TFR over the period 1917 to 1968 and the exponent is the adjustment from annual data to $T = 10$ and 25 year generations. We find $A/a = 1.14$ and $\varphi = 0.44$.

It follows from Proposition 4 that $n_{bt}(s_t) = \frac{(1-\varphi)(A/a)s_t - \varphi\pi}{\lambda}$ and $\overline{n_b} = E\left(n_{bt}\left(s_t\right)\right) = \frac{(1-\varphi)A/a-\varphi\pi}{\lambda}$. Thus fertility in the model expressed as percent deviations from the mean are

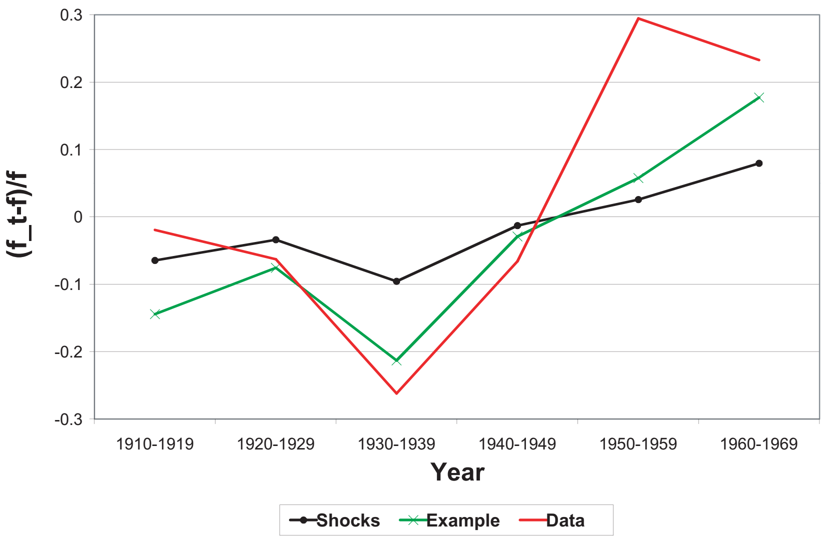
$$\begin{aligned} \frac{n_{bt}\left(s_t\right) - \overline{n_b}}{\overline{n_b}} &= \frac{(1-\varphi)A/a(s_t - 1)}{(1-\varphi)A/a - \varphi\pi} \\ &= 0.64(s_t - 1)/0.29 \end{aligned}$$

This implies that if $s_t = 0.9$, $\frac{n_{bt}(s_t) - \overline{n_b}}{\overline{n_b}} = -0.2$. That is, a 10 percent deviation from trend in TFP results in a 22 percent reduction in fertility – the elasticity of fertility with respect to a shock to productivity is about 2.

Figure 7 plots the percent deviations predicted by the model, relative to data and the sequence of shocks (i.e. $s_t - 1$, which corresponds to $\pi = 0$). As can be seen in the figure, for the 1930s, the model predicts a downswing in fertility of 21.3 percent compared to 26.2 percent in the data. It predicts a baby boom of 17.7 percent in the 1960s, while the largest increase (29.5 percent) actually happened in the 1950s.

The response in this case is larger than in the full depreciation case ($\pi = 0$). The intuition for this result is as follows. In the case where survival is zero, the dynasty has to rebuild the entire population every period. Labor being the only input in production, to smooth consumption, it cannot invest too little in children. If the stock of people does not die out every period, it is much easier to adjust today's fertility rate to keep consumption high today and still have relatively high expected consumption tomorrow.

Figure 7: Deviations from Trend in TFR: Model versus Data, Goods Cost



3.2.2 Time cost only ($\theta_t = bA(s_t)$)

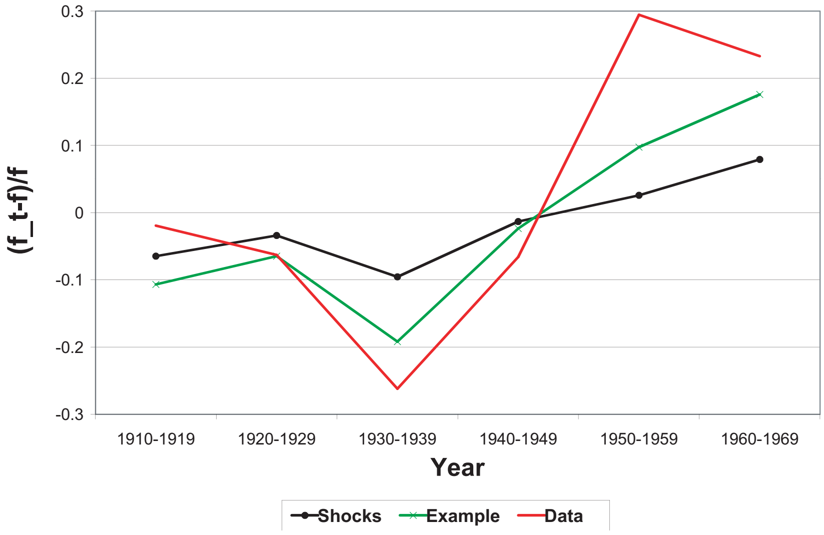
In this example, we set $b = 0.225$ (Boldrin et al. (2005)). This additional fact allows us to determine the intertemporal elasticity of substitution parameter, σ to match the average population growth rate, $E(n(s)) = 1.087$. We proceed as follows. First, we use value function iteration to solve for policy functions.[15] We then calculate $E(n(s))$ from the model and adjust σ, until $E(n(s)) = 1.087$. When $\pi = 0.8$, the calibration gives a value of $\sigma = 4.9$. This value is substantially higher than in the goods cost case and is partly responsible for the high elasticity. Mechanically speaking, to get an intertemporal elasticity of substitution similar to the one in the goods cost case (i.e. $\sigma = 2.1$) while still matching the average population growth of $E(n(s)) = 1.087$ per decade, the time cost of children has to be of the order of $b = 0.8$ per child. We further discuss sensitivity with respect to σ, β and

[15]Note that, in the time cost case, population growth and fertility are not linear functions of the shocks. See Appendix for details.

π in the Appendix.

We then feed in the observed sequence of shocks as before. Figure 8 plots the percent deviations predicted by the model, relative to data and the sequence of shocks $(s_t - 1)$. In this case, the effects of productivity shocks are slightly smaller than in the goods cost case with an elasticity of about 1.9.

Figure 8: Deviations from Trend in TFR: Model versus Data, Time Cost



4 Concluding remarks

In this paper we have analyzed a stochastic version of the dynastic model of endogenous fertility with preferences a la Barro-Becker. With a production function that employs only labor, we found that fertility does not depend on the stock of the population. Exploiting the analogy of our model to stochastic Ak models, we found that fertility is indeed pro-cyclical in most cases. Quantitatively, the predicted model responses of fertility to shocks in productivity are substantial particularly, when survival rates are realistically calibrated. The estimated elasticity lies between 1 and 2. In these simple models, productivity shocks capture about 70 percent of the pre-WWII baby bust in the U.S.. For the post-WWII baby boom, the predictions of this model are smaller and happen late compared to the data. Generally speaking, given the simplicity of the model, we view the quantitative results as quite a success.

There are several issues that are of interest by way of extension of the simple models laid out here so far. First, there are those that should be explored for the given state space of the model. These include considering the case with $\eta + \sigma$ different from 1, including autocorrelation in the shock in the model and alternative treatments of the data. We are currently exploring these variations of the model and data treatments.

In addition to these extensions, there are some that involve changing the state space. These changes may well alter the 'no catching-up' result – for instance, adding physical capital or land. This leads us back to Malthus and Easterlin. From the perspective of a theorist of economic growth, the Malthusian view is cast in terms of a 'stationary' model, one in which there is no persistent growth in income per capita but there is an essential fixed factor, e.g, land. In such a setting, summarized as $Y_t = F(L, N_t)$ where L is the time invariant stock of land and N_t is the time-varying population, standard neoclassical properties imply that when N_t is below (above) its stable long run level, N^*, the wage rate $w_t = F_N(L, N_t)$ increases above (below) its 'natural' level $w^* = F_N(L, N^*)$. Also, from a growth-theoretical perspective, the view exemplified by the work of Easterlin verbally describes a 'growth' model $Y_t = F(X_t, N_t)$, in which labor productivity $w_t = F_N(X_t, N_t)$ grows at a rate $\gamma_t = \rho\gamma_{t-1} + \varepsilon_t$, with ε_t a possibly random disturbance, and the 'other inputs' X_t are either not essential in production or fully reproducible. In general, this can either be an exogenous growth model, in which the growth rate γ_t is determined outside the model, or an endogenous one, in which γ_t is in fact a function of the rate at which the reproducible ones among the inputs X_t are accumulated. Either way, oscillations in population are mean reverting, to the long run *natural level* in the case of non-reproducible inputs and to the long run *natural growth rate* in the reproducible case, at least as long as the growth rate of income tends toward some long run average value.[16]

As a benchmark, in this paper, we have taken the simplest case, in which output is just equal to labor times productivity. Now, when the production function assumes that some other (essential) input is used in production together with labor, the final effect depends on whether the additional factor is fixed or accumulable. If the additional input is fixed, it tends to dampen the impact of a productivity shock on fertility because, everything else the same, an increase (decrease) in N_t decreases (increases) labor productivity in this case. To the contrary, when the additional input is easily accumulable, its presence may magnify the variation in fertility as the productivity of N_t receives a second boost, over and above the one coming from the original productivity shock, from the increase in the second input.

In Boldrin et al. (2006a), we use the intuition and results developed here, focus on adding physical capital to the model and apply this framework to cross-country differences in the size of depressions and baby busts, as well as war capital built-ups, destruction and deaths during WWII (summarized as a shock to the capital-labor ratio) and baby booms. Preliminary results show that cross-country correlations in sizes of depressions and baby busts as well as the correlation between WWII-related effects on capital-labor ratios and sizes of baby booms are large. In Boldrin et al. (2006b), we introduce land instead and use the model to analyze various historical episodes cast in a pre-industrial world. What we have not addressed so far are the long-run decrease in fertility as well as human capital as an additional input into production.

[16] Again, in both cases the crucial steps when taking the theory to the data consist in making additional operational assumptions about (a) what the long run growth rate (level) of income is, and (b) how economic agents make predictions about it, and the degree of time persistence in the deviations from the supposedly stable long run value. Our studies make no exception to this rule.

A Appendix

A.1 Proof of Proposition 4

Proof. From above the FOC's become:

$$(FOC1) \qquad N' = \left[\tfrac{\beta(1-\sigma)D}{a}\right]^{1/\sigma} C,$$

$$(FOC2) \qquad C + aN' = (As + a\pi)\, N.$$

Substituting $(FOC1)$ into $(FOC2)$, we obtain:

$$C + a\left[\tfrac{\beta(1-\sigma)D}{a}\right]^{1/\sigma} C = (As + a\pi)\, N, \text{ or,}$$

$$C\left[1 + a\left[\tfrac{\beta(1-\sigma)D}{a}\right]^{1/\sigma}\right] = (As + a\pi)\, N.$$

Thus,

$$C(N, s) = \varphi\,(As + a\pi)\, N, \text{ where,}$$

$$\varphi = \frac{1}{\left[1+a\left[\frac{\beta(1-\sigma)D}{a}\right]^{1/\sigma}\right]} = \frac{1}{\left[1+a^{\frac{\sigma-1}{\sigma}}(E(V(1,s'))\beta(1-\sigma))^{\frac{1}{\sigma}}\right]}.$$

It also follows that:

$$C + aN' = (As + a\pi)\, N, \text{ and so,}$$

$$N'(N, s) = \tfrac{(1-\varphi)(As+a\pi)}{a} N.$$

Thus, using this, we can characterize the Value Function by:

$$(BE) \qquad V(N, s) \equiv [C(N, s)]^{1-\sigma}/(1-\sigma) + \beta\,(N'(N, s))^{1-\sigma} D$$

$$= [\varphi\,(As + a\pi)\, N]^{1-\sigma}/(1-\sigma) + \beta\left[\tfrac{(1-\varphi)(As+a\pi)N}{a}\right]^{1-\sigma} D = B\,[(As + a\pi)\, N]^{1-\sigma},$$

where,

$$B = \left[\frac{\varphi^{1-\sigma}}{1-\sigma} + \beta \left[\frac{(1-\varphi)}{a} \right]^{1-\sigma} D \right] = \left[\frac{\varphi^{1-\sigma}}{1-\sigma} + \beta \left[\frac{1-\varphi}{a} \right]^{1-\sigma} E\left[V(1, s') \right] \right].$$

Thus, integrating, we obtain:

$$E\left[V(1, s) \right] = BE\left[(As + a\pi)^{1-\sigma} \right].$$

This, along with the other equations above and the definitions of φ and B gives a complete solution to the problem.

Further, $C = \varphi \left(As + a\pi \right) N$ and $N' = \frac{(1-\varphi)}{a} \left(As + a\pi \right) N$ where φ is as described in the Claim. □

A.2 Proof of Proposition 5

Proof. From above, the FOC's are:

$$(FOC1) \qquad N' = \left[\frac{\beta(1-\sigma)D}{bAs} \right]^{1/\sigma} C,$$

$$(FOC2) \qquad C + bAsN' = (1 + b\pi) AsN.$$

Substituting $(FOC1)$ into $(FOC2)$, we obtain:

$$C + bAs \left[\frac{\beta(1-\sigma)D}{bAs} \right]^{1/\sigma} C = (1 + b\pi) AsN, \text{ or,}$$

$$C \left[1 + bAs \left[\frac{\beta(1-\sigma)D}{bAs} \right]^{1/\sigma} \right] = (1 + b\pi) AsN.$$

Thus,

$$C(N, s) = \varphi(s) (1 + b\pi) AsN, \text{ where,}$$

$$\varphi(s) = \frac{1}{\left[1 + bAs \left[\frac{\beta(1-\sigma)D}{bAs} \right]^{1/\sigma} \right]} = \frac{1}{\left[1 + (bAs)^{1-1/\sigma} \left[\beta(1-\sigma) E[V(1, s')] \right]^{1/\sigma} \right]}.$$

It also follows that:

$$bAsN'(N, s) = (1 - \varphi(s)) (1 + b\pi) AsN, \text{ or,}$$

$$N'(N, s) = \frac{(1-\varphi(s))(1+b\pi)AsN}{bAs} = \frac{(1-\varphi(s))(1+b\pi)N}{b}.$$

□

B Sensitivity analysis

B.1 Details on the goods cost example

B.1.1 Technical notes

First we solve for φ and $A\_a \equiv \frac{A}{a}$, using only the above values. Then we use a normal distribution for the distribution of shocks, using the variance as calculated from detrended TFP data and reported in tfpdata3.doc. Then we solve for the preference parameter, σ, computationally. Without any additional fact, however, a and A cannot be determined separately. Details can be obtained from the authors upon request.

B.1.2 Benchmark calibration

The value of the utility parameter implied in this case is $\sigma = 2.11$ which compares quite well to the literature. Below we plot

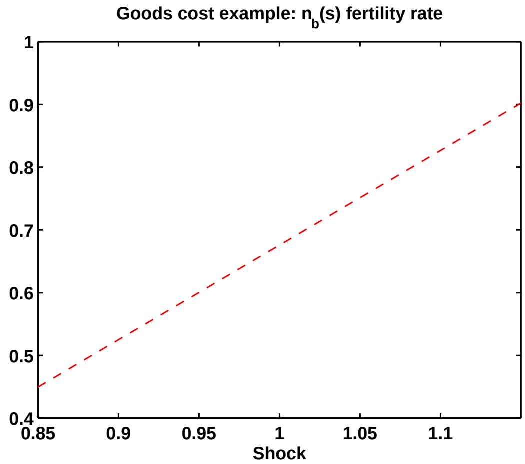
- $n_b(s)$, the fertility rate per fertile woman per decade[17],
- $n(s)$, the population growth rate and
- the value function.

Note that since $n_b(s) = \frac{n(s)-\pi}{\lambda} \geq 0$, population growth cannot fall below π due to the non-negativity constraint on fertility. The constraint binds for $s < 0.55$. The Great Depression of the 1930s has an estimated productivity shock of $s = 0.905$. Hence this constraint is unlikely to ever bind. Otherwise, fertility and population growth are linear in the shock.

Given our estimate for the standard deviation of shocks, $\sigma_s = 0.068$, the relevant range of shocks is $s \in [0.85, 1.15]$. We therefore plot a zoom of the previous figures to allow the reader to read off levels for the narrower range of shocks.

[17]The corresponding TFR would be about $3n_b$ if we assume women are fertile for 3 decades.

Figure 9: Goods cost example: fertility rate, $n_b(s)$



B.1.3 Sensitivity for the goods cost case (B stands for benchmark)

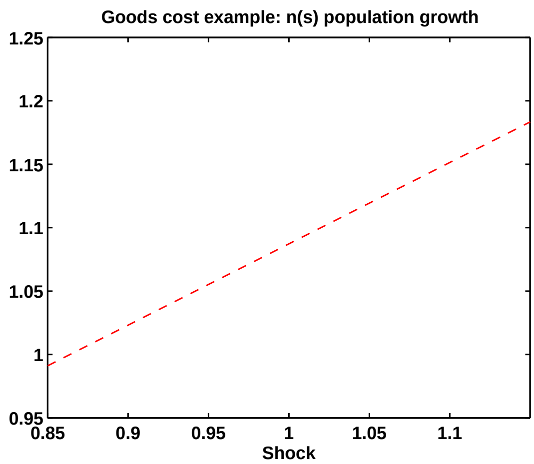
Next we present the effects from changing the deep parameters of the model, one at a time, without recalibrating, on $E(n(s))$ and the elasticity of fertility to productivity shocks. These are displayed in the Table below. We estimate the elasticity at the mean value and use a 10 percent deviation where we use the fact that fertility is affine in the shocks. Let $n_b(s) = a_p + b_p s$. Then we can write

$$\text{Elasticity} = \frac{\frac{n_b(s_t)-E(n_b(s_t))}{E(n_b(s_t))}}{s_t - E(s_t)} = \frac{\frac{n_b(0.9)-n_b(1)}{n_b(1)}}{0.9-1}$$
$$= -10 * \frac{n_b(0.9)-n_b(1)}{n_b(1)} = 10 * \frac{b_p(1-0.9)}{a_p+b_p} = \frac{b_p}{a_p+b_p}.$$

Higher values of σ imply that (aggregate) consumption smoothing is more important. That is, the planner likes to keep everything the same over time. One way of achieving this is to decrease population growth (column 2). Therefore, a_p decreases. This is an intuition from the non-stochastic case that continues to hold here. The stochastic feature of the model implies that b_p also decreases. Hence the elasticity increases.

Higher values of β holding σ constant imply a more patient planner. He

Figure 10: Goods cost example: population growth, $n(s)$



is therefore more willing to transfer consumption from today to tomorrow. To do that, he increases average population growth. This increases a_p and b_p. Thus the elasticity decreases.

Comparative statics with respect to π have partially been addressed in the text. As can be seen in the Table, average population growth increases with π. This is much like the average growth rate of the capital stock increasing when the depreciation rate decreases in the standard Ak model (this effect tends to increase a_p and b_p). However, as in the Ak case, this increase in growth rate does not require an increase in investment rates since a larger fraction of the capital stock survives from period to period. Indeed, investment as a fraction of output falls when the depreciation rate is lower. In the present model, the analogue of investment as a fraction of output is proportional to fertility $n_b(s) = \frac{A}{\lambda}\frac{N'(s)-\pi N}{AN}$. Because of this a_p is actually decreasing in π (this holds for $\sigma > 1$). b_p is also decreasing in π. Thus the elasticity is increasing in π.

Figure 11: Goods cost example: value function, $v(s)$

Table 1: Sensitivity for the Goods Cost example

Parameter value	**E(n(s))**	**Elasticity**	
$\sigma = 1.11$	1.361	1.26	
$\sigma = 2.11$	1.087	1.96	B
$\sigma = 3.11$	1.004	2.55	
$\beta = 0.95^{10}$	0.985	2.76	
$\beta = 0.97^{10}$	1.087	1.96	B
$\beta = 0.99^{10}$	1.197	1.56	
$\pi = 0.7$	1.061	1.6	
$\pi = 0.8$	1.087	1.96	B
$\pi = 0.9$	1.113	2.57	

B.2 Details on the time cost example

B.2.1 Technical notes

For this case, we use a slightly different procedure. First, we solve for the value function given the set of parameters obtained from the deterministic BGP equations (see calibrationBGP.xls). Second, we iterate over σ so that $E(n(s)) = \left(\frac{TFR}{2}\pi^f\right)^{\frac{10}{25}} = 1.087$. Details can be obtained from the authors upon request.

B.2.2 Benchmark calibration

The value of the utility parameter implied in this case is $\sigma = 4.87$ which is quite a bit higher than in the goods cost case. Below we plot

- $n_b(s)$, the fertility rate per fertile woman per decade[18],
- $n(s)$, the population growth rate and
- the value function.

Here the constraint binds for $s < 0.6$, which happens very rarely given the low variance of shocks. Note that both fertility and population growth are still quite close to linear in the shocks.

Given our estimate for the standard deviation of shocks, $\sigma_s = 0.068$, the relevant range of shocks is $s \in [0.85, 1.15]$. We therefore plot a zoom of the previous figures to allow the reader to read off levels for the narrower range of shocks.

B.2.3 Sensitivity for the time cost case (B stands for benchmark)

In this case, the qualitative intuitions from the goods cost case go through. However, quantitatively, average population growth is less sensitive to changes in σ, β and π when all costs are time costs. The elasticity is less sensitive to changes in σ and β but more sensitive to changes in π.

[18]The corresponding TFR would be about $3n_b$ if we assume women are fertile for 3 decades.

Figure 12: Time cost example: fertility rate, $n_b(s)$

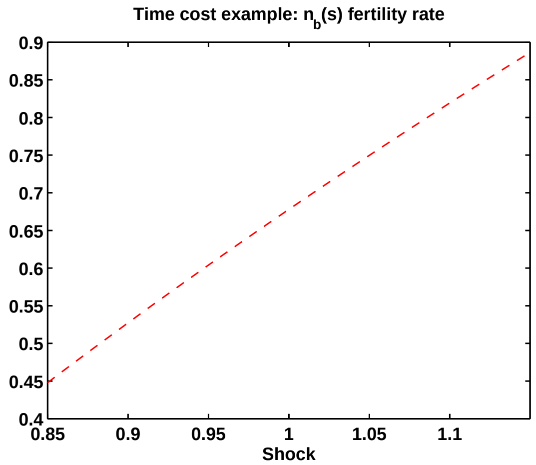


Figure 13: Time cost example: population growth, $n(s)$

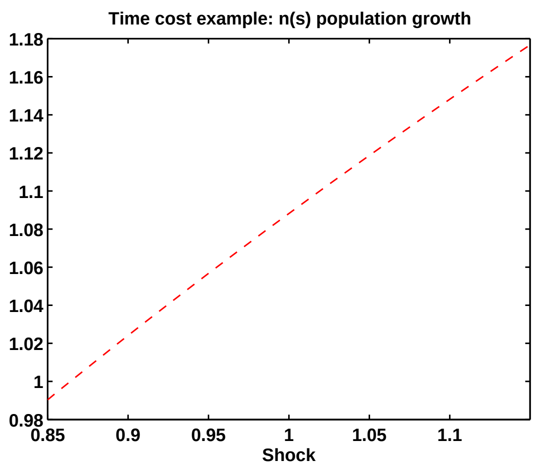


Figure 14: Time cost example: value function, $v(s)$

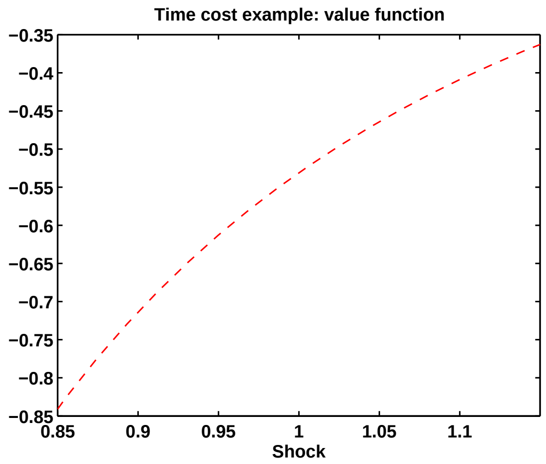


Table 2: Sensitivity for the Time Cost example

Parameter value	**E(n(s))**	**Elasticity**	
$\sigma = 2.87$	1.288	1.02	
$\sigma = 4.87$	1.087	1.92	B
$\sigma = 6.87$	1.015	2.64	
$\beta = 0.95^{10}$	1.042	2.22	
$\beta = 0.97^{10}$	1.087	1.92	B
$\beta = 0.99^{10}$	1.134	1.69	
$\pi = 0.7$	1.081	1.43	
$\pi = 0.8$	1.087	1.92	B
$\pi = 0.9$	1.093	2.89	

C Extensions of the AN model

C.1 Extensions holding the state space fixed

1.) Considering the case with $\eta + \sigma$ different from 1. This will not change some of the basic results – for example the result that there is no catching up. It might change some of the quantitative results however. It is unclear even the direction of these changes ex ante.

2.) Including autocorrelation in the shock in the model. Again, this will not change the negative catching up result. How important it is depends on the length of a period in the model. For example, the first order autocorrelation coefficient in the yearly data is about $\rho_1 = 0.86$, and assuming independence is fairly far off. On the other hand, the autocorrelation at 10 lags, is small ($\rho_{10} = 0.28$) and not statistically significantly different from 0. Thus, assuming independence of the shocks with a period of length 10 years is not such a bad assumption.

3.) Alternative treatments of the data. In the examples computed to this point, we have used a map between the model and data in which we have assumed that individual agents think that there is a common, exogenous, trend to TFP over the whole period of the data. The actual time series of the (log of) the Kendrick TFP data is shown in Figure 10, below. Thus, although it is not obviously insane to view this series as having one, common trend over the period, it would also be reasonable to model the series as one with lower growth near the beginning of the period, and higher growth toward the end of the sample. Various alternatives would interesting to explore in more detail. For example, a version of the model where there are two, still exogenous, growth rates for TFP for the two periods, γ_1 and γ_2, with $\gamma_2 > \gamma_1$, with higher frequency fluctuations around these trends, would be an interesting generalization to explore. There are three (at least) serious conceptual issues that will arise. First, it is not clear, from visual inspection, when this 'change' should occur. Was the break in the early 1920s? Or did it occur later, during WWII for example? This affects the size of the shocks (and even the sign in some cases). Second, did the individual decision-makers in the model know that the break occurred? Or did they learn it, slowly, over time? Is it safe or reasonable to model this change as one that was viewed, ex ante, as a zero probability event? Is it safe or reasonable to model the corresponding opposite change (i.e., the event of growth returning to γ_1 after the change) as one that was viewed, ex post, as a zero probability event? These issues will make undertaking this venture challenging, but still doable.

C.2 Extensions enlarging the state space

1.) Add a vintage structure to the lifetimes of the dynasty, i.e., have realistic lifetimes, and overlapping members of the dynasty alive at each time. One advantage of this is that we can then try to address the question: What birth cohorts have low (high) fertility when a negative (positive) productivity shock occurs? How does this compare with which cohorts actual had low (high) fertility during the bust (boom)?

A simple version of this would be something like:

$$P(N_0, s_0) \qquad Max_{\{C_t, N_t\}} \qquad U_0(\{C_t, N_t\}) = E_0 \left[\sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} \frac{C_t^{1-\sigma}}{1-\sigma} \right]$$

subject to:

$$C_t + \sum_{a=1}^{a_L-1} \theta_{at} N_{at} + \theta_{0t} N_{bt+1} \leq W_t, \qquad W_t = \sum_{a=a_L}^{T} N_{at} p_a w_t,$$

$$N_t = \sum_{a=a_L}^{T} N_{at}, \qquad N_{a+1t+1} = \pi_a N_{at},$$

$$N_{1t} = \pi_1 N_{bt}, \qquad (N_{00}, N_{10}, ..., N_{T0}) \text{ given.}$$

In such a setting λ_t, the fraction of fertile people in the population, may not be independent of time anymore. From Proposition 2, this suggests that catching up may occur.

2.) For extensions in which the production function includes either physical capital or land, see Boldrin et al. (2006a) and Boldrin et al. (2006b) and the conclusion of this paper.

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