# Property Rights and Efficiency in OLG Models with Endogenous Fertility

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# Property rights over offspring's labor income

Who can legally (and feasibly) make decisions about
a person/child as a resource?

- the parents? the person/child? the government?
- Clearly, a person cannot decide to be born.
- Laws and cultural norms determine property rights
  - mandatory parental support;
  - parent's control over offspring;
  - allocation of power between generations.

Allocation of property rights matters for fertility choice.

Missing market $\rightarrow$ fertility inefficiently low.

# An extreme example

Measure 1 of potential parents live for 2 periods & solve:

$$\max_{c^m, c^o, n, s} \quad U^P = \ln(c^m) + \beta \ln(c^o) \quad \text{s.t.} \quad c^m + \theta n + s = e, \quad c^o = rs.$$

Children in period 2, if alive, solve:

$$\max_{c^k} \quad U^k = \ln(c^k) \quad \text{s.t.} \quad c^k = w.$$

Production in period 2:

$$Y = K^\alpha L^{1-\alpha} \quad \rightarrow \quad r = \alpha k^{\alpha-1}, \quad w = (1-\alpha)k^\alpha, \quad k = K/L.$$

Market clearing: $K = s$ and $L = n$.

In equilibrium, $n = 0$ because child cost = $\theta$, benefit = 0.

$$\Rightarrow \quad Y = 0 \quad \Rightarrow \quad c^o = 0 \quad \Rightarrow \quad U^P = -\infty.$$

# An extreme example

Measure 1 of potential parents live for 2 periods & solve:

$$\max_{c^m, c^o, n, s} \quad U^P = \ln(c^m) + \beta \ln(c^o) \quad \text{s.t.} \quad c^m + \theta n + s = e, \quad c^o = rs + \omega n.$$

Children in period 2, if alive, solve:

$$\max_{c^k} \quad U^k = \ln(c^k) \quad \text{s.t.} \quad c^k = w - \omega.$$

Production in period 2:

$$Y = K^\alpha L^{1-\alpha} \quad \rightarrow \quad r = \alpha k^{\alpha-1}, \quad w = (1-\alpha)k^\alpha, \quad k = K/L.$$

Market clearing: $K = s$ and $L = n$.

In equilibrium, $n > 0$ because return to children $\frac{\omega}{\theta} > 0$.

Choice of $s$ and $n$ adjusts s.t. $k = K/L = s/n$ gives $r = \omega/\theta$.

If $U^k = \ln(w - \omega) > u(unborn)$, even children are better off.

# What we do

1. From extreme example:
   - If property rights (PR) lie with children:
     fertility inefficiently low,
     i.e. fertility up, through PR shift $\rightarrow$ everyone better off.
2. Analyze general OLG model:
   - children are a consumption good,
   - parents are altruistic,
   - property rights: minimum transfer constraint,
   - appropriate efficiency concept: $\mathcal{A}-$efficiency

   **Main result:** Equilibrium allocation $\mathcal{A}-$efficient
   $\Leftrightarrow$ parents are not transfer constrained.
3. Revisit previous efficiency results in OLG.
4. Policy implications

# What we do

3. Revisit previous efficiency results in OLG.

| | exogenous fertility | endogenous fertility |
|---|---|---|
| no altruism | Samuelson (1958),<br>Cass (1972),<br>Balasko and Shell (1980)<br>$(r > n)$ nec. & suff. for PO | Michel, Wigniolle (2007),<br>Conde-Ruiz, Giménez and<br>Pérez-Nievas (2004)<br>$(r > n)$ not suff. for $\mathcal{M}-$efficiency<br>$(r > w/\theta)$ suff. for $\mathcal{M}-$efficiency |
| with altruism | Barro (1974),<br>Burbidge (1983)<br>“operative transfers”<br>nec. & suff for PO | Pazner and Razin (1979)<br>$(r > n)$ always, “efficient” |

## This Paper:

- Non-altruistic models implicity assume children have PR. Altruistic models often implicitly assume parents have PR.
- Property rights: key dimension not analyzed before!

# What we do

1. Extreme example.
2. Analyze general OLG model.
3. Revisit previous efficiency results in OLG.
4. Policy implications:
    - PAYG pension:
      relaxes transfer constraint, does not lead to efficiency,
    - Alternative 1: Fertility dependent PAYG,
    - Alternative 2: Fertility subsidy and Government debt.

# The Model

Households:

$$\max_{c_t^m, n_t, c_{t+1}^o, s_{t+1}, \{b_{t+1}^i\}_i} \quad U_t = u(c_t^m) + \beta u(c_{t+1}^o) + V\left(n_t, \frac{\int_0^{n_t} U_{t+1}^i di}{n_t}\right)$$

$$\begin{aligned}
s.t. \quad & c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t) \\
& c_{t+1}^o + \int_0^{n_t} b_{t+1}^i w_{t+1} di \leq r_{t+1} s_{t+1} \\
& b_{t+1}^i \geq \underline{b}_{t+1} \\
& c_t^m, c_{t+1}^o, n_t \geq 0
\end{aligned}$$

# The Model

Households:

$$\max_{c_t^m, n_t, c_{t+1}^o, s_{t+1}, b_{t+1}} \quad U_t = u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1})$$

$$s.t. \quad c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t)$$

$$c_{t+1}^o + n_t b_{t+1} w_{t+1} \leq r_{t+1} s_{t+1}$$

$$b_{t+1} \geq \underline{b}_{t+1}$$

$\underline{b}_{t+1}$ can be interpreted as property rights:

- $\underline{b}_{t+1} = -1$ parents own children's income
- $\underline{b}_{t+1} = 0$ children own their own income

# The Model

## Households:

$$\max_{c_t^m, n_t, c_{t+1}^o, s_{t+1}, b_{t+1}} \quad U_t = u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1})$$

$$\begin{aligned} s.t. \quad & c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t) \\ & c_{t+1}^o + n_t b_{t+1} w_{t+1} \leq r_{t+1} s_{t+1} \\ & b_{t+1} \geq \underline{b}_{t+1} \end{aligned}$$

**Production:**

$$\begin{aligned} Y_t &= F(K_t, L_t) \\ w_t &= F_L(k_t, 1) \\ r_t &= F_K(k_t, 1) \end{aligned}$$

**Mkts clear:**

$$\begin{aligned} L_t &= n_{t-1} \\ K_t &= s_t = k_t n_{t-1} \end{aligned}$$

(Note: full depreciation)

# Costs and Benefits of Child-rearing

$$V_n(n_t, U_{t+1}) = \beta u'(c^o_{t+1})\Big[r_{t+1}\theta_t + b_{t+1}w_{t+1}\Big]$$

The higher $\underline{b}_{t+1}$, the more likely constraint is binding,

$\rightarrow$ increases cost of children,

$\rightarrow$ distorts incentive to have children.

Equalizing inter-temporal and -generational MU in cons.:

$$\begin{aligned} u'(c^m_t) &= \beta u'(c^o_{t+1})r_{t+1} \\ \beta u'(c^o_{t+1})n_t &= V_U(n_t, U_{t+1})u'(c^m_{t+1}) + \frac{\lambda_{b,t+1}}{w_{t+1}} \end{aligned}$$

$\lambda_{b,t+1}$: how far off most preferred consumption allocation?

# Utility Specifications

$$U_t = u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1})$$

Razin-Ben-Zion (*RB*) specification given by:

$$V(n_t, U_{t+1}) = \gamma u(n_t) + \zeta U_{t+1}$$

Barro-Becker type altruism (*BB*) given by:

$$V(n_t, U_{t+1}) = \zeta g(n_t) U_{t+1}$$

For $u(\cdot) = \log(\cdot)$, *RB* and *BB* represent the same preferences.

Generally, the two are not a special case of each other.

# Optimal Transfer
# $\underline{b} = -1$

Assume: $u(\cdot) = \log(\cdot)$, $\zeta < 1$, $\gamma > \zeta(1 + \gamma + \beta) > 0$.

$$b^* = \frac{\theta r^* \zeta(1 + \beta + \gamma) - w^* \gamma}{w^* (\gamma - \zeta(1 + \gamma + \beta))}$$

Note:

- $b^*$ may be negative – even with altruism.
- Especially if $\zeta$ small, $\gamma$ large, $w$ high or $r$ low.
- Suggests that even altruistic parents want to "steal"/take from their children in many circumstances.

# $\mathcal{A}-$ and $\mathcal{P}-$Efficiency

Golosov, Jones and Tertilt (2007)

**Definition**

A feasible allocation is $\mathcal{A}-$efficient if there is no other feasible allocation such that all people *alive* under both allocations are no worse off and at least one is strictly better off.

**Definition**

A feasible allocation is $\mathcal{P}-$efficient if there is no other feasible allocation such that all *potential* people are no worse off and at least one is strictly better off. (*)

[(*)Note: requires a utility function that is defined over states of the world where a person is not born.]

# $\mathcal{A}-$ and $\mathcal{P}-$Efficiency: Results

### Proposition

*Assume $V_U > 0$. If parameters are such that $\lambda_{b,t} = 0$ for all $t$, then the equilibrium allocation, $z^* \equiv \{c_t^{m*}, c_{t+1}^{o*}, n_t^*, s_{t+1}^*, k_t^*, b_{t+1}^*\}_{t=0}^{\infty}$, is $\mathcal{A}-$ (and $\mathcal{P}-$) efficient.*

### Proposition

*Assume $V_U > 0$. If parameters are such that $\lambda_{b,s+1} > 0$ for some generation $s$, then the equilibrium allocation, $\hat{z} \equiv \{\hat{c}_t^m, \hat{c}_{t+1}^o, \hat{n}_t, \hat{s}_{t+1}, \hat{k}_t, \hat{b}_{t+1}\}_{t=0}^{\infty}$, is $\mathcal{A}-$ (and $\mathcal{P}-$) inefficient.*

# $\mathcal{A}-$superior allocation to $\hat{z}$

Generation $s$ receives:

$$\tilde{c}_s^m = \hat{c}_s^m - \theta_s \epsilon \qquad \tilde{n}_s = \hat{n}_s + \epsilon$$
$$\tilde{c}_{s+1}^o = \hat{c}_{s+1}^o + (\Delta - \underline{b}_{s+1}\hat{w}_{s+1})\epsilon \qquad \tilde{s}_{s+1} = \hat{s}_{s+1}.$$

$\epsilon-$mass of *new* people (not alive in $\hat{z}$) receive:

$$\tilde{c}_{s+1}^{m,n} = \frac{F(\hat{s}_{s+1}, \tilde{n}_s) - F(\hat{s}_{s+1}, \hat{n}_s)}{\epsilon} - \hat{s}_{s+2} - \theta_{s+1}\hat{n}_{s+1} + \underline{b} - \Delta$$
$$\tilde{c}_{s+1}^{o,n} = \hat{c}_{s+1}^o \qquad \tilde{n}_{s+1}^n = \hat{n}_{s+1} \qquad \tilde{s}_{s+2}^n = \hat{s}_{s+2}$$

Note: $\lim_{\epsilon \to 0} \tilde{c}_{s+1}^{m,n} = \hat{c}_{s+1}^m - \Delta$.

Everyone else receives the same as in $\hat{z}$.

Note: Feasible by construction.

# $\mathcal{A}-$superior allocation to $\hat{z}$

Allocation $\mathcal{A}-$ and $\mathcal{P}-$superior:

- Generation $s$:

$$\frac{\frac{\partial \tilde{U}_s(\epsilon,\Delta)}{\partial \epsilon}\Big|_{\epsilon=0}}{\partial \Delta}\Big|_{\Delta=0} = \beta u'(\hat{c}^o_{s+1}) - V_U\left(\hat{n}_s, \hat{U}_{s+1}\right)\frac{u'(\hat{c}^m_{s+1})}{\hat{n}_s}$$
$$= \frac{\lambda_{b,s+1}}{\hat{n}_s} > 0$$

Hence, $\exists \epsilon, \Delta > 0$ s.t. $\tilde{U}_s > \hat{U}_s$.

- For people alive in $\hat{z}$ and $t < s$: $\widetilde{U}_t > \hat{U}_t$ since $V_u > 0$
- For people alive in $\hat{z}$ and $t > s$: $\widetilde{U}_t = \hat{U}_t$
- $\epsilon-$mass new people: $\widetilde{U}^n_{s+1} > u(unborn)$.

# Efficiency Results and Coase's Theorem

## Coase's Theorem

Property rights don't matter for efficiency of allocation
—if bargaining is possible.

## Our results

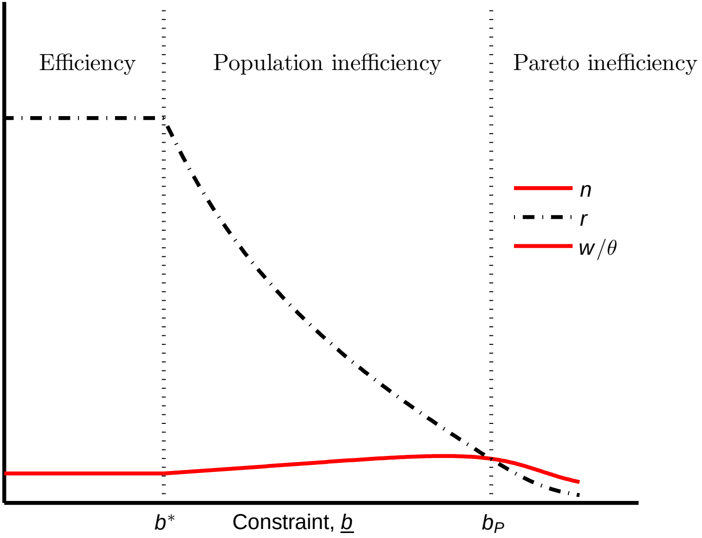
1. When parents "own" children, costs and benefits of having children borne by same people: parents.
   $\rightarrow$ equilibrium fertility is efficient
2. When parents don't "own" children, costs and benefits of having children borne by different people. Parents bear cost, children reap benefits.
   $\rightarrow$ equilibrium fertility not efficient
3. Unborn children cannot write contract with parents when property rights are assigned to them by law.

# $\mathcal{A}-$efficiency and Pareto efficiency

## Proposition

- *A stationary equilibrium allocation is Pareto efficient if and only if $r > n$.*
- *In a stationary equilibrium, $r > n$ is a necessary but not sufficient condition for $\mathcal{A}-$efficiency.*

# Steady State Efficiency Results



# What we do

| | exogenous fertility | endogenous fertility |
|---|---|---|
| no altruism | Samuelson (1958), Cass (1972), Balasko and Shell (1980)<br>$(r > n)$ nec. & suff. for PO | Michel, Wigniolle (2007), Conde-Ruiz, Giménez and Pérez-Nievas (2004)<br>$(r > n)$ not suff. for $\mathcal{M}-$efficiency<br>$(r > w/\theta)$ suff. for $\mathcal{M}-$efficiency |
| with altruism | Barro (1974), Burbidge (1983)<br>"operative transfers" nec. & suff for PO | Pazner and Razin (1979)<br>$(r > n)$ always, "efficient" |

## This Paper:

$(r > n)$ necessary but not sufficient for $\mathcal{A}-$efficiency

$(r > w/\theta)$ necessary but not sufficient for $\mathcal{A}-$efficiency

# $\mathcal{A}-$ and $\mathcal{P}-$Efficiency without Altruism

### Proposition

*Assume $V_U = 0$. Then the transfer constraint is always binding.*

*There are two cases:*

a) *if $\underline{b} > -1$, then the equilibrium is $\mathcal{A}-$ (and $\mathcal{P}-$) inefficient;*

b) *if $\underline{b} = -1$, then the equilibrium is such that*

   $c_t^m = c_{t+1}^o = n_{t-1} = 0$ *for all* $t \geq 1$,

   *and the equilibrium is $\mathcal{A}-$ (and $\mathcal{P}-$) efficient.*

$\Rightarrow$ Using $\mathcal{A}-$ or $\mathcal{P}-$efficiency

not very interesting in models without altruism

# Millian Efficiency

## Definition

A symmetric feasible allocation is $\underline{\mathcal{M}-\text{ efficient}}$ if there is no other symmetric feasible allocation such that all generations are no worse off and at least one generation is strictly better off.

Used by

Michel, Wigniolle (2007),

Conde-Ruiz, Giménez and Pérez-Nievas (2009)

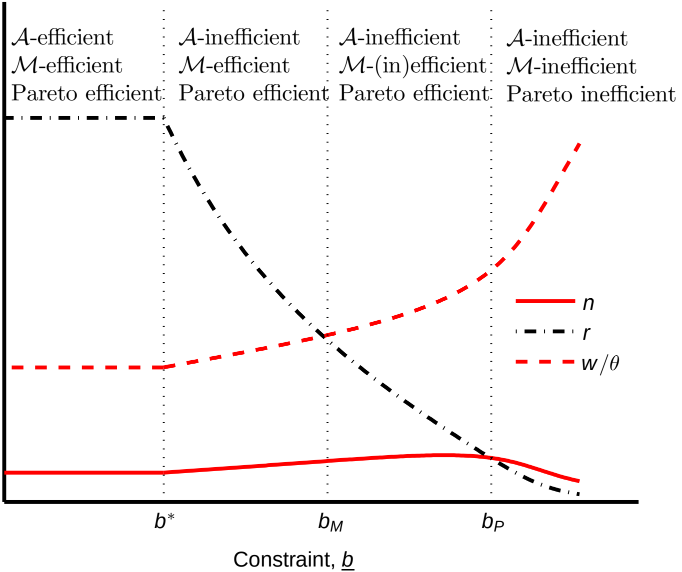
Under what conditions can $\hat{z}$ be dominated by a symmetric allocation?

# $\mathcal{A}-$efficiency and Millian efficiency

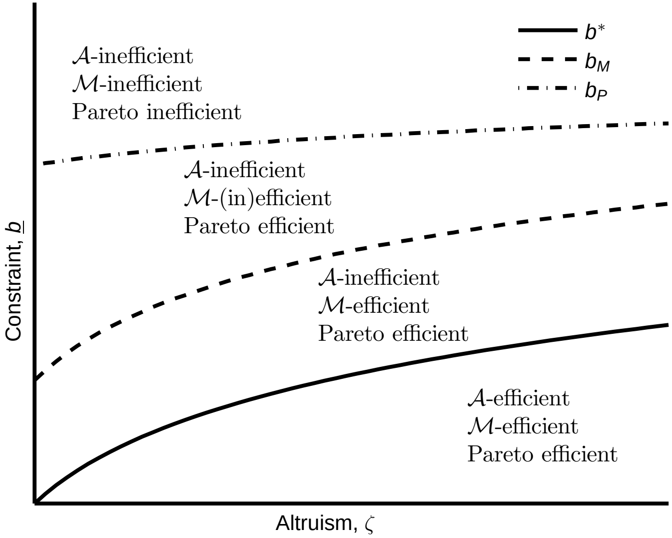
## Proposition

- *A stationary equilibrium allocation is $\mathcal{M}-$efficient if $r\theta > w$.*
- *In a stationary equilibrium, $r\theta > w$ is a necessary but not sufficient condition for $\mathcal{A}-$efficiency.*

# Steady State Efficiency Results



# Property Rights versus Altruism



# Policy Implications

1. The introduction of standard PAYG pensions
   - alleviates downward pressure on fertility (at first);
   - relaxes transfer constraint;
   - equilibrium allocation NOT $\mathcal{A}-$efficient.
2. Alternative I: Fertility dependent PAYG pensions (FDPAYG)
   - alleviates downward pressure on fertility;
   - aligns costs and benefits of having children;
   - equilibrium allocation $\mathcal{A}-$efficient.
3. Alternative II: Fertility subsidy and Government debt
   - same as FDPAYG

# PAYG Pension System

Households:

$$\begin{aligned} \max U_t =& u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1}) \\ s.t. \quad & c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t - \tau_t) \\ & c_{t+1}^o + b_{t+1} w_{t+1} n_t \leq r_{t+1} s_{t+1} + T_{t+1} \\ & b_{t+1} \geq \underline{b}_{t+1} \end{aligned}$$

Gov.ment budget balance: $T_t = n_{t-1} \tau_t w_t$

Note: Since labor supply is inelastic, $\tau_t$ proportional but still lump-sum for period $t$.

# Efficiency of PAYG Pension System?

Budget constraint:

$$c^o_{t+1}+[c^m_{t+1}+\theta_{t+1}n_{t+1}+s_{t+2}-w_{t+1}+\tau_{t+1}w_{t+1}]n_t \leq r_{t+1}s_{t+1}+T_{t+1}$$

- Lump-sum taxes (per person) are not really lump-sum!
- They distort fertility decision (more children = more taxes).
- Parent does not realize that more children also increase $T_{t+1}$.
- Even if constraint not binding: Fertility inefficiently low.

$\Rightarrow$ “Operative transfers” not sufficient with fertility choice

# What we do

| | exogenous fertility | endogenous fertility |
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| no altruism | Samuelson (1958),<br>Cass (1972),<br>Balasko and Shell (1980)<br>$(r > n)$ nec. & suff. for PO | Michel, Wigniolle (2007),<br>Conde-Ruiz, Giménez and Pérez-Nievas (2004)<br>$(r > n)$ not suff. for $\mathcal{M}-$efficiency<br>$(r > w/\theta)$ suff. for $\mathcal{M}-$efficiency |
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## This Paper:

Operative transfers necessary but not sufficient for $\mathcal{A}-$efficiency.

# Alternative I: Pay-out depends on $n$

## $T(n_t) = n_t \tau_{t+1} w_{t+1}$

Households:

$$\begin{aligned} \max U_t =& u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1}) \\ s.t. \quad & c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t - \tau_t) \\ & c_{t+1}^o + b_{t+1} w_{t+1} n_t \leq r_{t+1} s_{t+1} + \tau_{t+1} w_{t+1} n_t \\ & b_{t+1} \geq \underline{b}_{t+1} \end{aligned}$$

- Note that $b$ and $\tau$ enter symmetrically.
  $\rightarrow$ increase $\tau$ increases $b^*$ one for one
- Choose $\tau$ s.t. $b^* \geq \underline{b}$ not binding.
- Allocation is $\mathcal{A}-$efficient.
- Aligns costs and benefits of child-rearing.

# Alternative II: Fertility subsidy and Government debt

## Households:

$$\max U_t = u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1})$$

$$s.t. \quad c_t^m + \theta_t n_t + (s_{t+1} + d_{t+1}) \leq w_t(1 + b_t - \tau_t^d) + \tau_t^s n_t$$

$$c_{t+1}^o + b_{t+1} w_{t+1} n_t \leq r_{t+1}(s_{t+1} + d_{t+1})$$

$$b_{t+1} \geq \underline{b}_{t+1}$$

**Gov.ment budget:** $n_{t-1}(d_{t+1} + \tau_t^d w_t) = r_t d_t + \tau_t^s n_t n_{t-1}$

Set $\tau_t^d = \tau_t$.

Set $\tau_t^s = \tau_{t+1} \frac{w_{t+1}}{r_{t+1}}$.

$\rightarrow$ same solution as FDPAYG, with $d_{t+1} = \tau_t^s n_t$.

"Ricardian Equivalence"

# Summary

- Many countries are worried about ‘too low fertility’.
- We provide a rationale for pronatalist policies.
- Misaligned property rights lead to inefficiently low fertility.

  $\longrightarrow$ Coase’s Theorem.

  $\longrightarrow$ Property rights and Efficiency in OLG.
- PAYG pensions:
  1. Alleviates downward pressure on fertility
  2. Distorts fertility decision.
  3. Alternatives: Fertility dependent PAYG or<br>Fertility subsidy and Gov debt

What’s next?

# What's next?

- Analogy investment in children's human capital
- Why did property rights shift from parents to children?
  - Political economy of shift in property rights?
    - Who wanted to pass laws and why?
    - Who was constrained?
  - Technological reasons?
    - rural vs urban, extended vs nuclear families?
- Quantitative importance?
  - How much of a contribution to fertility history in the US?
    - Average decrease, boom and bust? Differential fertility?
  - Which countries experience(ed) inefficiently low fertility?
    - Welfare gains from policy reform?

# Adding Human Capital

- Parents cannot borrow against children's income and resulting inefficiencies in human capital investment
  $\rightarrow$ pointed out before in the literature.
- Fernandez and Rogerson (2001),
  Aiyagari, Greenwood, Seshadri (2002),
  Boldrin and Montes (2005), ...
- Focus in literature:
  borrowing constraints in *exogenous* fertility context.

next

# Analogy: Fertility and Human Capital decisions

- Both $e$ and $n$ are inefficiently low when constraint binding.
- One critical difference:

  costs and benefits of HK investments aligned if *child* makes decisions and credit markets function.
- *Not* possible for fertility decisions

  – a child can never decide to be born!

next

# What's next?

- Analogy investment in children's human capital
- Why did property rights shift from parents to children?
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