Property Rights and Efficiency in OLG Models with Endogenous Fertility

Alice Schoonbroodt$^1$  Michèle Tertilt$^2$

$^1$U. of Southampton and CPC
$^2$Stanford U., U. of Mannheim, NBER and CEPR

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Property rights over offspring’s labor income

Who can legally (and feasibly) make decisions about a person/child as a resource?

- the parents? the person/child? the government?
- Clearly, a person cannot decide to be born.
- Laws and cultural norms determine property rights
  - mandatory parental support;
  - parent’s control over offspring;
  - allocation of power between generations.

Allocation of property rights matters for fertility choice.

Missing market → fertility inefficiently low.
An extreme example

Measure 1 of potential parents live for 2 periods & solve:

$$\max_{c^m, c^o, n, s} U^P = \ln(c^m) + \beta \ln(c^o) \quad \text{s.t.} \quad c^m + \theta n + s = e,$$

$$c^o = rs.$$

Children in period 2, if alive, solve:

$$\max_{c^k} U^k = \ln(c^k) \quad \text{s.t.} \quad c^k = w.$$

Production in period 2:

$$Y = K^\alpha L^{1-\alpha} \quad \rightarrow \quad r = \alpha k^{\alpha - 1}, \quad w = (1 - \alpha)k^\alpha, \quad k = K/L.$$

Market clearing: $K = s$ and $L = n$.

In equilibrium, $n = 0$ because child cost = $\theta$, benefit = 0.

$$\Rightarrow \quad Y = 0 \quad \Rightarrow \quad c^o = 0 \quad \Rightarrow \quad U^P = -\infty.$$
An extreme example

Measure 1 of potential parents live for 2 periods & solve:

$$\max_{c^m, c^o, n, s} U^P = \ln(c^m) + \beta \ln(c^o) \quad \text{s.t.} \quad c^m + \theta n + s = e,$$

$$c^o = rs + \omega n.$$  

Children in period 2, if alive, solve:

$$\max_{c^k} U^k = \ln(c^k) \quad \text{s.t.} \quad c^k = w - \omega.$$  

Production in period 2:

$$Y = K^\alpha L^{1-\alpha} \quad \rightarrow \quad r = \alpha k^{\alpha-1}, \quad w = (1 - \alpha) k^\alpha, \quad k = K/L.$$  

Market clearing: $K = s$ and $L = n.$

In equilibrium, $n > 0$ because return to children $\frac{\omega}{\theta} > 0.$

Choice of $s$ and $n$ adjusts s.t. $k = K/L = s/n$ gives $r = \omega/\theta.$

If $U^k = \ln(w - \omega) > u(\text{unborn}),$ even children are better off.
1. From extreme example:
   - If property rights (PR) lie with children: fertility inefficiently low,
     i.e. fertility up, through PR shift $\rightarrow$ everyone better off.

2. Analyze general OLG model:
   - children are a consumption good,
   - parents are altruistic,
   - property rights: minimum transfer constraint,
   - appropriate efficiency concept: $\mathcal{A}$-efficiency

   **Main result:** Equilibrium allocation $\mathcal{A}$-efficient
   $\iff$ parents are not transfer constrained.

3. Revisit previous efficiency results in OLG.

4. Policy implications
3. Revisit previous efficiency results in OLG.

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This Paper:

- Non-altruistic models implicitly assume children have PR.
  Altruistic models often implicitly assume parents have PR.
- Property rights: key dimension not analyzed before!
What we do

1. Extreme example.
2. Analyze general OLG model.
3. Revisit previous efficiency results in OLG.
4. Policy implications:
   - PAYG pension:
     - relaxes transfer constraint, does not lead to efficiency,
   - Alternative 1: Fertility dependent PAYG,
   - Alternative 2: Fertility subsidy and Government debt.
The Model

Households:

$$\max_{c^m_t, n_t, c^o_{t+1}, s_{t+1}, \{b^i_{t+1}\}_i} U_t = u(c^m_t) + \beta u(c^o_{t+1}) + V \left( n_t, \frac{\int_0^{n_t} U_{t+1}^i \, di}{n_t} \right)$$

s.t.

$$c^m_t + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t)$$

$$c^o_{t+1} + \int_0^{n_t} b^i_{t+1} w_{t+1} \, di \leq r_{t+1} s_{t+1}$$

$$b^i_{t+1} \geq b^i_{t+1}$$

$$c^m_t, c^o_{t+1}, n_t \geq 0$$
The Model

Households:

\[
\max_{c_t^m, n_t, c_{t+1}^o, s_{t+1}, b_{t+1}} 
U_t = u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1})
\]

s.t.

\[
c_t^m + \theta_t n_t + s_{t+1} \leq w_t (1 + b_t) \\
c_{t+1}^o + n_t b_{t+1} w_{t+1} \leq r_{t+1} s_{t+1} \\
b_{t+1} \geq b_{t+1}
\]

\(b_{t+1}\) can be interpreted as property rights:

- \(b_{t+1} = -1\) parents own children’s income
- \(b_{t+1} = 0\) children own their own income
The Model

Households:

\[
\max_{c_t^m, n_t, c_{t+1}^o, s_{t+1}, b_{t+1}} U_t = u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1})
\]

s.t.
\[
c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t) \\
c_{t+1}^o + n_t b_{t+1} w_{t+1} \leq r_{t+1} s_{t+1} \\
b_{t+1} \geq b_{t+1}
\]

Production:
\[
Y_t = F(K_t, L_t) \\
w_t = F_L(k_t, 1) \\
r_t = F_K(k_t, 1)
\]

Mkts clear:
\[
L_t = n_{t-1} \\
K_t = s_t = k_t n_{t-1}
\]

(Note: full depreciation)
Costs and Benefits of Child-rearing

\[ V_n(n_t, U_{t+1}) = \beta u'(c_{t+1}^o) \left[ r_{t+1} \theta_t + b_{t+1} w_{t+1} \right] \]

The higher \( b_{t+1} \), the more likely constraint is binding,

→ increases cost of children,
→ distorts incentive to have children.

Equalizing inter-temporal and -generational MU in cons.:

\[ u'(c_t^m) = \beta u'(c_{t+1}^o) r_{t+1} \]

\[ \beta u'(c_{t+1}^o) n_t = V_U(n_t, U_{t+1}) u'(c_{t+1}^m) + \frac{\lambda_{b,t+1}}{w_{t+1}} \]

\( \lambda_{b,t+1} \): how far off most preferred consumption allocation?
Utility Specifications

\[ U_t = u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1}) \]

Razin-Ben-Zion (RB) specification given by:

\[ V(n_t, U_{t+1}) = \gamma u(n_t) + \zeta U_{t+1} \]

Barro-Becker type altruism (BB) given by:

\[ V(n_t, U_{t+1}) = \zeta g(n_t) U_{t+1} \]

For \( u(\cdot) = \log(\cdot) \), RB and BB represent the same preferences.

Generally, the two are not a special case of each other.
Optimal Transfer

\[ b = -1 \]

Assume: \( u(\cdot) = \log(\cdot), \zeta < 1, \gamma > \zeta (1 + \gamma + \beta) > 0. \)

\[
\begin{align*}
b^* &= \frac{\theta r^* \zeta (1 + \beta + \gamma) - w^* \gamma}{w^* (\gamma - \zeta (1 + \gamma + \beta))}
\end{align*}
\]

Note:

- \( b^* \) may be negative – even with altruism.
- Especially if \( \zeta \) small, \( \gamma \) large, \( w \) high or \( r \) low.
- Suggests that even altruistic parents want to “steal”/take from their children in many circumstances.
A– and P–Efficiency

Golosov, Jones and Tertilt (2007)

Definition
A feasible allocation is \( A–\)efficient if there is no other feasible allocation such that all people \( alive \) under both allocations are no worse off and at least one is strictly better off.

Definition
A feasible allocation is \( P–\)efficient if there is no other feasible allocation such that all \( potential \) people are no worse off and at least one is strictly better off. (*)

[(*):Note: requires a utility function that is defined over states of the world where a person is not born.]
\( \mathcal{A} \)– and \( \mathcal{P} \)–Efficiency: Results

Proposition

Assume \( V_U > 0 \). If parameters are such that \( \lambda_{b,t} = 0 \) for all \( t \), then the equilibrium allocation,
\[
z^* \equiv \{c_{m,t}^*, c_{o,t+1}^*, n_t^*, s_{t+1}^*, k_t^*, b_{t+1}^*\}_{t=0}^\infty,
\]
is \( \mathcal{A} \)– (and \( \mathcal{P} \)–) efficient.

Proposition

Assume \( V_U > 0 \). If parameters are such that \( \lambda_{b,s+1} > 0 \) for some generation \( s \), then the equilibrium allocation,
\[
\hat{z} \equiv \{\hat{c}_{m,t}, \hat{c}_{o,t+1}, \hat{n}_t, \hat{s}_{t+1}, \hat{k}_t, \hat{b}_{t+1}\}_{t=0}^\infty,
\]
is \( \mathcal{A} \)– (and \( \mathcal{P} \)–) inefficient.
A—superior allocation to \( \hat{z} \)

Generation \( s \) receives:

\[
\tilde{c}_s^m = \hat{c}_s^m - \theta_s \epsilon \quad \tilde{n}_s = \hat{n}_s + \epsilon
\]
\[
\tilde{c}_{s+1}^o = \hat{c}_{s+1}^o + (\Delta - b_{s+1} \hat{w}_{s+1}) \epsilon \quad \tilde{s}_{s+1} = \hat{s}_{s+1}.
\]

\( \epsilon \)—mass of new people (not alive in \( \hat{z} \)) receive:

\[
\tilde{c}_{s+1}^{m,n} = \frac{F(\hat{s}_{s+1}, \tilde{n}_s) - F(\hat{s}_{s+1}, \hat{n}_s)}{\epsilon} - \hat{s}_{s+2} - \theta_{s+1} \hat{n}_{s+1} + b - \Delta
\]
\[
\tilde{c}_{s+1}^{o,n} = \hat{c}_{s+1}^o \quad \tilde{n}_{s+1}^n = \hat{n}_{s+1} \quad \tilde{s}_{s+2}^n = \hat{s}_{s+2}
\]

Note: \( \lim_{\epsilon \to 0} \tilde{c}_{s+1}^{m,n} = \hat{c}_{s+1}^m - \Delta \).

Everyone else receives the same as in \( \hat{z} \).

Note: Feasible by construction.
\( \mathcal{A} \)-superior allocation to \( \hat{z} \)

Allocation \( \mathcal{A}- \) and \( \mathcal{P}- \)superior:

- **Generation \( s \):**

  \[
  \frac{\partial \tilde{U}_s(\epsilon, \Delta)}{\partial \epsilon} \bigg|_{\epsilon=0} \bigg|_{\Delta=0} = \beta u'(\hat{c}_{s+1}^o) - V_u(\hat{n}_s, \hat{U}_{s+1}) \frac{u'(\hat{c}_{s+1}^m)}{\hat{n}_s} \\
  = \frac{\lambda_{b,s+1}}{\hat{n}_s} > 0
  \]

  Hence, \( \exists \epsilon, \Delta > 0 \) s.t. \( \tilde{U}_s > \hat{U}_s \).

- **For people alive in \( \hat{z} \) and \( t < s \):** \( \tilde{U}_t > \hat{U}_t \) since \( V_u > 0 \)

- **For people alive in \( \hat{z} \) and \( t > s \):** \( \tilde{U}_t = \hat{U}_t \)

- **\( \epsilon \)-mass new people:** \( \tilde{U}_{s+1}^n > u(unborn) \).
Efficiency Results and Coase’s Theorem

Coase’s Theorem
Property rights don’t matter for efficiency of allocation
— if bargaining is possible.

Our results

1. When parents “own” children, costs and benefits of having children borne by same people: parents.
   → equilibrium fertility is efficient

   → equilibrium fertility not efficient

3. Unborn children cannot write contract with parents when property rights are assigned to them by law.
A—efficiency and Pareto efficiency

Proposition

- A stationary equilibrium allocation is Pareto efficient if and only if $r > n$.

- In a stationary equilibrium, $r > n$ is a necessary but not sufficient condition for A—efficiency.
Steady State Efficiency Results

Efficiency  Population inefficiency  Pareto inefficiency

$\frac{w}{\theta}$

Constraint, $b$

$\star$

$P$
## What we do

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**This Paper:**

\((r > n)\) necessary but not sufficient for \(\mathcal{A}\)–efficiency

\((r > w/\theta)\) necessary but not sufficient for \(\mathcal{A}\)–efficiency
Proposition
Assume $V_U = 0$. Then the transfer constraint is always binding.

There are two cases:

a) if $b > -1$, then the equilibrium is $A^-$ (and $P^-$) inefficient;

b) if $b = -1$, then the equilibrium is such that

$$c_t^m = c_t^o = n_{t-1} = 0 \text{ for all } t \geq 1,$$

and the equilibrium is $A^-$ (and $P^-$) efficient.

$\Rightarrow$ Using $A^-$ or $P^-$-efficiency
not very interesting in models without altruism
Millian Efficiency

Definition
A symmetric feasible allocation is \( M \)-efficient if there is no other symmetric feasible allocation such that all generations are no worse off and at least one generation is strictly better off.

Used by
- Michel, Wigniolle (2007),
- Conde-Ruiz, Giménez and Pérez-Nievas (2009)

Under what conditions can \( \hat{z} \) be dominated by a symmetric allocation?
A–efficiency and Millian efficiency

Proposition

- A stationary equilibrium allocation is $\mathcal{M}$–efficient if $r\theta > w$.
- In a stationary equilibrium, $r\theta > w$ is a necessary but not sufficient condition for $A$–efficiency.
Steady State Efficiency Results

A-efficient  A-inefficient  A-inefficient  A-inefficient
M-efficient  M-efficient  M-(in)efficient  M-inefficient
Pareto efficient  Pareto efficient  Pareto efficient  Pareto inefficient

Constraint, $b$

$n$  
$r$  
$w/\theta$
Property Rights versus Altruism

\[ A\text{-inefficient} \]
\[ M\text{-inefficient} \]
\[ \text{Pareto inefficient} \]

\[ A\text{-inefficient} \]
\[ M\text{-(in)efficient} \]
\[ \text{Pareto efficient} \]

\[ A\text{-efficient} \]
\[ M\text{-efficient} \]
\[ \text{Pareto efficient} \]
Policy Implications

1. The introduction of standard PAYG pensions
   ▶ alleviates downward pressure on fertility (at first);
   ▶ relaxes transfer constraint;
   ▶ equilibrium allocation NOT $A-$efficient.

2. Alternative I: Fertility dependent PAYG pensions (FDPAYG)
   ▶ alleviates downward pressure on fertility;
   ▶ aligns costs and benefits of having children;
   ▶ equilibrium allocation $A-$efficient.

3. Alternative II: Fertility subsidy and Government debt
   ▶ same as FDPAYG
PAYG Pension System

Households:

\[
\begin{aligned}
\text{max } U_t &= u(c_t^m) + \beta u(c_{t+1}^o) + V(n_t, U_{t+1}) \\
\text{s.t. } &c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t - \tau_t) \\
&c_{t+1}^o + b_{t+1} w_{t+1} n_t \leq r_{t+1} s_{t+1} + T_{t+1} \\
&b_{t+1} \geq b_{t+1}
\end{aligned}
\]

Gov.ment budget balance: \( T_t = n_{t-1} \tau_t w_t \)

Note: Since labor supply is inelastic, \( \tau_t \) proportional but still lump-sum for period \( t \).
Efficiency of PAYG Pension System?

Budget constraint:

\[ c_{t+1}^o + \left[ c_{t+1}^m + \theta_{t+1} n_{t+1} + s_{t+2} - w_{t+1} + \tau_{t+1} w_{t+1} \right] n_t \leq r_{t+1} s_{t+1} + T_{t+1} \]

- Lump-sum taxes (per person) are not really lump-sum!
- They distort fertility decision (more children = more taxes).
- Parent does not realize that more children also increase \( T_{t+1} \).
- Even if constraint not binding: Fertility inefficiently low.

⇒ “Operative transfers” not sufficient with fertility choice
What we do

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| no altruism   | Samuelson (1958), Cass (1972), Balasko and Shell (1980) 
(r > n) not suff. for $\mathcal{M}$-efficiency 
(r > w/θ) suff. for $\mathcal{M}$-efficiency |
“operative transfers” 
nec. & suff for PO | Pazner and Razin (1979) 
(r > n) always, “efficient” |

This Paper:

Operative transfers necessary but not sufficient for $\mathcal{A}$-efficiency.
Alternative I: Pay-out depends on $n$

$$T(n_t) = n_t \tau_{t+1} w_{t+1}$$

Households:

$$\text{max } U_t = u(c^m_t) + \beta u(c^o_{t+1}) + V(n_t, U_{t+1})$$

Subject to:

$$c^m_t + \theta_t n_t + s_{t+1} \leq w_t (1 + b_t - \tau_t)$$

$$c^o_{t+1} + b_{t+1} w_{t+1} n_t \leq r_{t+1} s_{t+1} + \tau_{t+1} w_{t+1} n_t$$

$$b_{t+1} \geq b_{t+1}$$

- Note that $b$ and $\tau$ enter symmetrically.
  → increase $\tau$ increases $b^*$ one for one
- Choose $\tau$ s.t. $b^* \geq b$ not binding.
- Allocation is $\mathcal{A}$-efficient.
- Aligns costs and benefits of child-rearing.
Alternative II: Fertility subsidy and Government debt

Households:

\[
\max U_t = u(c^m_t) + \beta u(c^o_{t+1}) + V(n_t, U_{t+1})
\]

\[
s.t. \quad c^m_t + \theta_t n_t + (s_{t+1} + d_{t+1}) \leq w_t(1 + b_t - \tau^d_t) + \tau^s_t n_t
\]

\[
c^o_{t+1} + b_{t+1} w_{t+1} n_t \leq r_{t+1}(s_{t+1} + d_{t+1})
\]

\[
b_{t+1} \geq b_{t+1}
\]

Government budget:

\[
n_{t-1}(d_{t+1} + \tau^d_t w_t) = r_t d_t + \tau^s_t n_t n_{t-1}
\]

Set \( \tau^d_t = \tau_t \).

Set \( \tau^s_t = \tau_{t+1} \frac{w_{t+1}}{r_{t+1}} \).

→ same solution as FDPAYG, with \( d_{t+1} = \tau^s_t n_t \).

“Ricardian Equivalence”
Many countries are worried about ‘too low fertility’.

We provide a rationale for pronatalist policies.

Misaligned property rights lead to inefficiently low fertility.

→ Coase’s Theorem.

→ Property rights and Efficiency in OLG.

PAYG pensions:

1. Alleviates downward pressure on fertility
2. Distorts fertility decision.
3. Alternatives: Fertility dependent PAYG or Fertility subsidy and Gov debt

What’s next?
What’s next?

- Analogy investment in children’s human capital

- Why did property rights shift from parents to children?
  - Political economy of shift in property rights?
    - Who wanted to pass laws and why?
    - Who was constrained?
  - Technological reasons?
    - rural vs urban, extended vs nuclear families?

- Quantitative importance?
  - How much of a contribution to fertility history in the US?
    - Average decrease, boom and bust? Differential fertility?
  - Which countries experience(ed) inefficiently low fertility?
    - Welfare gains from policy reform?
Adding Human Capital

- Parents cannot borrow against children’s income and resulting inefficiencies in human capital investment → pointed out before in the literature.


- Focus in literature: borrowing constraints in *exogenous* fertility context.
Analogy: Fertility and Human Capital decisions

- Both $e$ and $n$ are inefficiently low when constraint binding.
- One critical difference:
  costs and benefits of HK investments aligned if child makes decisions and credit markets function.
- *Not* possible for fertility decisions
  – a child can never decide to be born!
What’s next?

- Analogy investment in children’s human capital
- Why did property rights shift from parents to children?
  - Political economy of shift in property rights?
    - Who wanted to pass laws and why?
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