# Baby Busts and Baby Booms

## The Fertility Response to Shocks in Dynastic Models

Larry Jones[1] Alice Schoonbroodt[2]

[1]University of Minnesota and NBER

[2]University of Southampton and CPC

DGEM, REDg at CEMFI
September 2010

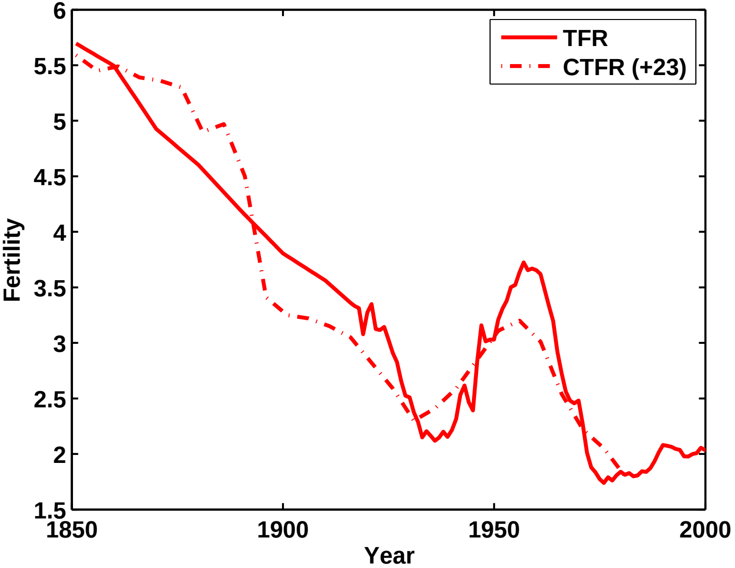
# Motivation

Large fluctuations in fertility during the 20th century

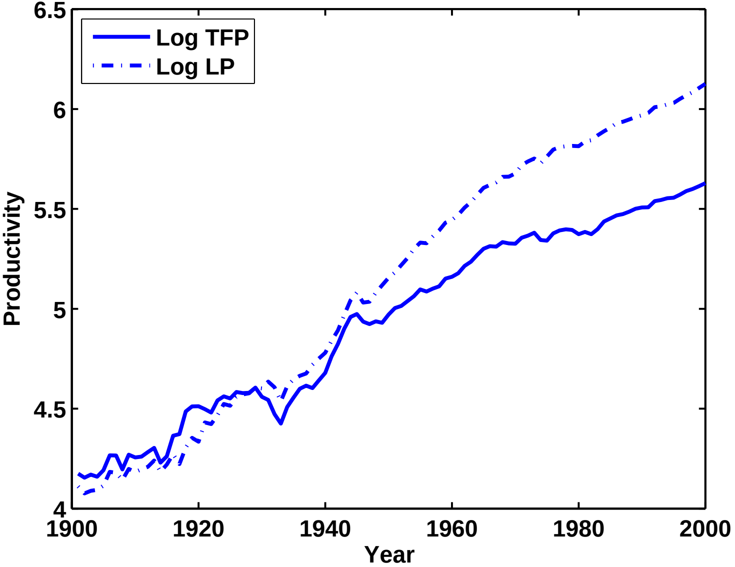
- In the U.S.
- In other developed countries – sizes differ
  $\rightarrow$ Large: U.S., Canada, Australia
  $\rightarrow$ Smaller: European countries

$\rightarrow$ Demographers: link fertility fluctuations to G-D, WWII,...
(good vs. bad times, optimism, pessimism, **catching up**,...)

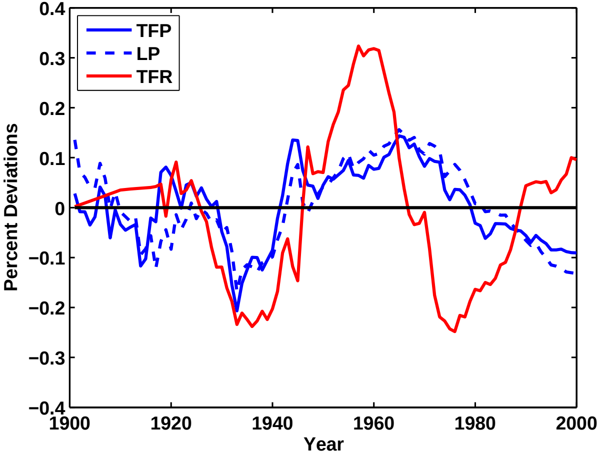
# U.S. TFR and CTFR 1850-1990



# Empirical Evidence: TFP and LP 1900-2000



# Empirical Evidence: % Deviations in TFP&LP and TFR




$$\widehat{TFR}_t = -0.0051 + 0.836\widehat{TFP}_t - 0.84\widehat{TFP}_{t-20} + \varepsilon_t$$

# Motivation

Large fluctuations in fertility during the 20th century

$\longrightarrow$ Demographers: link fertility fluctuations to G-D, WWII,...

(good vs. bad times, optimism, pessimism, catching up,...)

Economic interpretation of demographer's story:

$\longrightarrow$ Easterlin (1961,...,2000)

Neo-classical environments:

$\longrightarrow$ Greenwood, Seshadri, Vandenbroucke (2005)

$\longrightarrow$ Doepke, Hazan, Maoz (2007)

$\longrightarrow$ Albanesi, Olivetti (2009)

$\longrightarrow$ ...

# This Paper

## Model

- Stochastic neo-classical growth model
  *Population $N$ plays the role of capital $K$*
  *Different ages of people similar to capital vintages*
- Non-stochastic growth model with endogenous fertility
  *Children cost time $\rightarrow$ opportunity cost pro-cyclical*
  *Children enter utility function $\rightarrow$ smooth like consumption*

## Cyclical properties?

## Current fertility dependent on past fertility?

## Quantitative experiments: Std. Recession & U.S. BBB

## Cross-country: Sizes of G-D, Baby Busts and Booms

# Preview of Results

- Qualitatively:
  - Fertility pro-cyclical in most cases

    depends on nature of costs of children

    $\longrightarrow$ all goods: pro-cyclical

    $\longrightarrow$ all time: consumption smoothing $\rightarrow$ pro-cyclical
    opportunity cost $\rightarrow$ counter-cyclical
  - Current fertility depends negatively on last period's fertility

    except if only one period of working life
- Quantitatively, interesting magnitudes

  $\longrightarrow$ Standard Recession: not much on CTFR, maybe TFR

  $\longrightarrow$ Depression in 1930s: Baby Bust

  Baby Bust + high productivity: Baby Boom 1950s

  Continuing fluctuations dampened by productivity shocks
- International Evidence: size of Depression, Baby Bust and Boom

# Barro-Becker model with Age-Structure

Preferences

$$U_t^1 = V_t^1 + \beta g(n_t) U_{t+1}^1$$

where $V_t^1 = \sum_{a=1}^{3} \beta^{a-1} u(c_{t+a-1}^a)$

Laws of motion

$N_t^1 = n_{t-1} N_{t-1}^1$ is the number of births in period $t-1$

$N_t^a = N_{t-1}^{a-1}$ for $a = 2, 3$

$N_t^a = 0$ for $a > 3$

$N_0^3 = 1$

Sequential substitution and $g(n) = n^\eta$

Preferences of Dynastic Head (initial old)

$$U_0^3 = \sum_{t=0}^{\infty} \beta^t \left[ \sum_{a=1}^{3} g(N_t^a) u(c_t^a) \right]$$

# Barro-Becker model with Age-Structure

Feasibility for Dynasty: $\forall t$

$$\sum_{a=1}^{3} N_t^a c_t^a + \theta_t(w_t^1) n_t N_t^1 \leq \sum_{a=1}^{2} w_t^a N_t^a$$

Introduce Productivity shocks

$$(w_t^1, w_t^2) = (\gamma^t s_t w^1, \gamma^t s_t w^2)$$

$E(s_t) = 1$ and $s_t$ are $i.i.d.$

Costs of children

Goods cost: $\theta_t(w_t^1) = \gamma^t \theta$

Time cost: $\theta_t(w_t^1) = b w_t^1 = b \gamma^t s_t w^1$

[DEC]

# Dynastic Planner's Problem, $P(\gamma, \beta; \{N_0^a\}, s_0)$

$$\max E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \sum_{a=1}^{3} g(N_t^a(s^{t-a})) u(c_t^a(s^t)) \right] |s_0 \right]$$

subject to:

$$\sum_{a=1}^{3} N_t^a(s^{t-a}) c_t^a(s^t) + \theta(s^t) N_{t+1}^1(s^t) \leq \gamma^t s_t \sum_{a=1}^{2} w^a N_t^a(s^{t-a});$$

$N_{t+1}^1(s^t)$ is the number of births in period $t$;
$N_t^a(s^{t-a}) = N_{t-1}^{a-1}(s^{t-1-(a-1)})$ for $a = 2, 3$;
$N_t^a(s^{t-a}) = 0$ for $a > 3$;
$N_0^a$ given, $a = 1, 2, 3$.

where $s^t = (s_0, s_1, ...., s_t)$ is the history of shocks.

Notice non-convexity $N_t^a c_t^a \to C_t^a$.

# Dynastic Planner's Problem, $P(1, \beta\gamma^{1-\sigma}; \{N_0^a\}, s_0)$

$$\max E_0 \left[ \sum_{t=0}^{\infty} \left(\beta\gamma^{1-\sigma}\right)^t \left[ \sum_{a=1}^{3} g\left(N_t^a(s^{t-a})\right) u\left(\frac{C_t^a(s^t)}{N_t^a(s^{t-a})}\right)\right] |s_0 \right]$$

subject to:

$$\sum_{a=1}^{3} C_t^a(s^t) + \theta(s^t) N_{t+1}^1(s^t) \leq s_t \sum_{a=1}^{2} w^a N_t^a(s^{t-a});$$

$N_{t+1}^1(s^t)$ is the number of births in period $t$;
$N_t^a(s^{t-a}) = N_{t-1}^{a-1}(s^{t-1-(a-1)})$ for $a = 2, 3$;
$N_t^a(s^{t-a}) = 0$ for $a > 3$;
$N_0^a$ given, $a = 1, 2, 3$.

where $C_t^a$ and $\theta_t$ are detrended.

# Preliminary results to simplify DP

Assume $\beta\gamma^{1-\sigma} < 1$.

Assume $g(N) = N^{\eta}$ and $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$

Parameter restrictions for monotonicity and concavity:

1.) B-B: $0 < 1 - \sigma \leq \eta < 1$, or,

2.) G&BC: $\eta \leq 1 - \sigma < 0$

Let $V(N^1, N^2, N^3; s)$ be maxed value in $P(1, \beta\gamma^{1-\sigma}; \left\{N_0^a\right\}, s_0)$

Then, $V(N^1, N^2, N^3; s)$ is homog. of d$^o$ $\eta$ in $(N^1, N^2, N^3)$.

# Dynastic Planner's Problem, $P(1, \beta\gamma^{1-\sigma}; \{N_0^a\}, s_0)$

$$\max E_0 \left[ \sum_{t=0}^{\infty} \left(\beta\gamma^{1-\sigma}\right)^t \left[ \sum_{a=1}^{3} \left(N_t^a(s^{t-a})\right)^{\eta} \frac{\left(\frac{C_t^a(s^t)}{N_t^a(s^{t-a})}\right)^{1-\sigma}}{1-\sigma} \right] \Big| s_0 \right]$$

subject to:

$$\sum_{a=1}^{3} C_t^a(s^t) + \theta(s^t) N_{t+1}^1(s^t) \leq s_t \sum_{a=1}^{2} w^a N_t^a(s^{t-a});$$

$N_{t+1}^1(s^t)$ is the number of births in period $t$;
$N_t^a(s^{t-a}) = N_{t-1}^{a-1}(s^{t-1-(a-1)})$ for $a = 2, 3$;
$N_t^a(s^{t-a}) = 0$ for $a > 3$;
$N_0^a$ given, $a = 1, 2, 3$.

where $C_t^a$ and $\theta_t$ are detrended.

# Preliminary results to simplify DP

Assume $\beta\gamma^{1-\sigma} < 1$.

Assume $g(N) = N^{\eta}$ and $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$

1.) B-B: $0 < 1 - \sigma \leq \eta < 1$, or,

2.) G&BC: $\eta \leq 1 - \sigma < 0$

Then, $V(N^1, N^2, N^3; s)$ is homog. of d$^o$ $\eta$ in $(N^1, N^2, N^3)$.

Assume $\eta = 1 - \sigma$. Then,

- $C^a \equiv c^a N^a = c^{a^*} N^{a^*} \equiv C^{a*}, \forall a, a^*$
- $N^3$ irrelevant $\Rightarrow$ use $\tilde{V}(N^1, N^2; s)$.

Homogeneity implies

$\tilde{V}(N^1, N^2; s) = (N^2)^{1-\sigma} \tilde{V}(N^1/N^2, 1; s) \equiv (N^2)^{1-\sigma} \widehat{V}(n; s)$.

# Bellman equation and FOC

Bellman equation where $n = N^1/N^2$, last period's fertility:

$$\widehat{V}(n;s) \equiv \max_{n'} \left[ \frac{\left(\frac{s\left[w^1 n + w^2\right] - \theta(s) n' n}{3}\right)^{1-\sigma}}{1-\sigma} + \beta\gamma^{1-\sigma} n^{1-\sigma} E\left[\widehat{V}(n';s')\right] \right]$$

The FOC is given by:

$$(FOC) \qquad \frac{\theta(s)}{3E\left[\hat{V}_1(n',s')\right]} = \beta\gamma^{1-\sigma}\left(\frac{s\left[w^1 + \frac{w^2}{n}\right] - \theta(s)n'}{3}\right)^{\sigma}$$

# Bellman equation and FOC

Bellman equation where $n = N^1/N^2$, last period's fertility:

$$\widehat{V}(n;s) \equiv \max_{n'} \left[ \frac{\left(\frac{s\left[w^1 n + w^2\right] - \theta(s) n' n}{3}\right)^{1-\sigma}}{1-\sigma} + \beta\gamma^{1-\sigma} n^{1-\sigma} E\left[\widehat{V}(n';s')\right] \right]$$

The FOC is given by:

$$(FOC) \qquad \frac{\theta(s)}{3E\left[\hat{V}_1(n',s')\right]} = \beta\gamma^{1-\sigma}\left(\frac{s\left[w^1 + \frac{w^2}{n}\right] - \theta(s)n'}{3}\right)^{\sigma}$$

# First-order Condition

Goods cost ($\theta(sw^1) = \theta$):

$$\frac{\theta}{3E\hat{V}_1(n', s')} = \beta\gamma^{1-\sigma}\left(\frac{s\left[w^1 + \frac{w^2}{n}\right] - \theta n'}{3}\right)^{\sigma}$$

Time cost ($\theta(sw^1) = bsw^1$):

$$\frac{bw^1 s^{1-\sigma}}{3E\hat{V}_1(n', s')} = \beta\gamma^{1-\sigma}\left(\frac{w^1(1 - bn') + \frac{w^2}{n}}{3}\right)^{\sigma}$$

### Proposition

*Current fertility, $n'(n; s)$ is*

1. a. *pro-cyclical if $\theta(s) = \theta$;*

   b. *pro-cyclical if $\theta(s) = bsw^1$ and $\sigma > 1$;*

   *counter-cyclical if $\theta(s) = bsw^1$ and $\sigma < 1$;*

2. a. *independent of the last period's fertility, $n$, if $w^2 = 0$;*

   b. *decreasing in last period's fertility, $n$ if $w^2 > 0$.*

*Thus, if $\sigma \neq 1$ and $w^2 > 0$ the model generates endogenous cycles, triggered by productivity shocks.*

[QUANT]

# Quantitative Experiments

Model period: 20 years, i.e. adult at age 20, fertile 20-40,
worker 20-60, retired 60-80

Calibration (to averages)

- Parameters for stochastic process ($\log \hat{s}_t \sim N(0, \sigma_s)$)
- Parameters for economic model ($\sigma, \beta, w^1, w^2, \gamma, \theta$ or $b$)

Experiments

- Impulse response $\rightarrow$ Typical recession
- Historical simulation:
  - Input: sequence of shock realizations, data $\rightarrow$ model
  - Output: sequence of fertility fluctuations, model $\rightarrow$ data

  Baseline + Sensitivityyyyy...

International (statistical) evidence

# Parameterization

Preset or set directly from wage and TFP data:

| Param. | $\sigma$ | $\beta$ | $w^1$ | $w^2$ | $\gamma$ | $\sigma_s$ |
|---|---|---|---|---|---|---|
| Value | 3.00 | $0.96^{20}$ | 1.00 | 1.25 | $1.016^{20}$ | 0.07 |

We experiment with 2 extreme cases: all goods vs. all time cost

Calibrated to match 0.645% annual population growth:

| Param. | $\theta$ (goods cost) | $b$ (time cost) |
|---|---|---|
| Value | 0.1932 | 0.1927 |

# Impulse Response and Recessions

A 1% increase in productivity, $s$, generates:

- Goods Cost: a 1.7% contemporaneous increase in fertility,
  a 1.6% decrease 1 period later
- Time Cost: : a 1.0% contemporaneous increase in fertility,
  a 0.9% decrease 1 period later

$\approx$ “Standard Recession” (e.g., 5% below trend for 2 years)

These are total effects on completed fertility: not that large.

**Great Depression:** 12% below trend for 10 years...

# How to fit model shocks to data shocks?

Model period is 20 years, 4 age groups, $a = 0, 1, 2, 3$.

Hence, fertile period should be 20 years, age 20 to 40.

Hence, relevant income shock should be over 20 year period.

However, this assumes that fertility is uniform age 20-40.

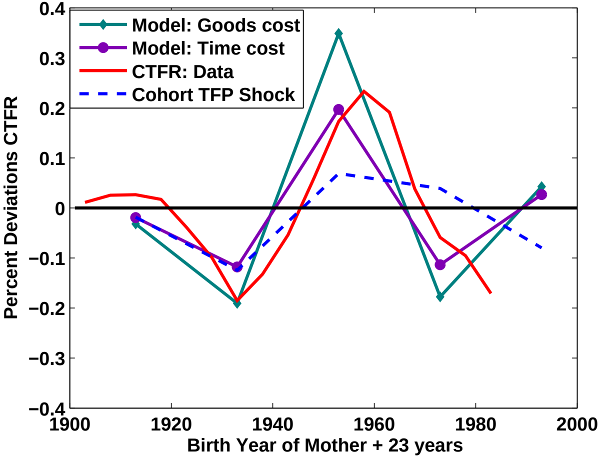
But 60% of all births occur between age 20-30, 75% age 20-35.

In particular, women age 20-30 in the 1920s are mostly done with fertility decisions by the time the Great Depression hits.

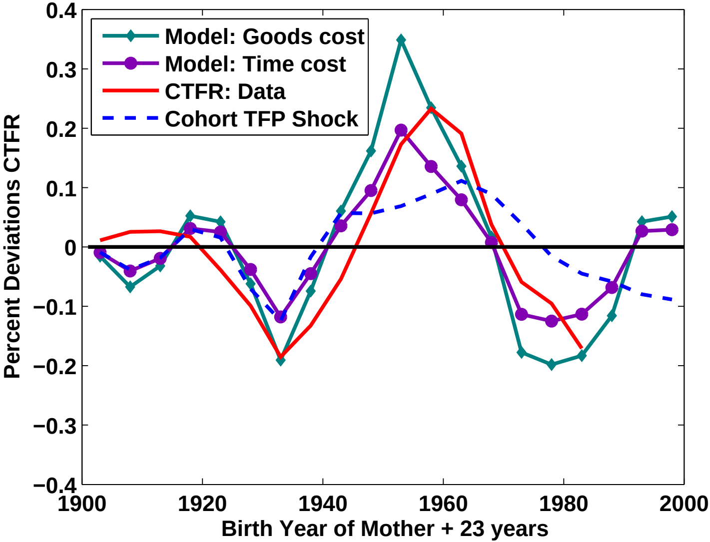
We therefore use 10 year period for shocks in baseline

# CTFR: Most affected Dynasty

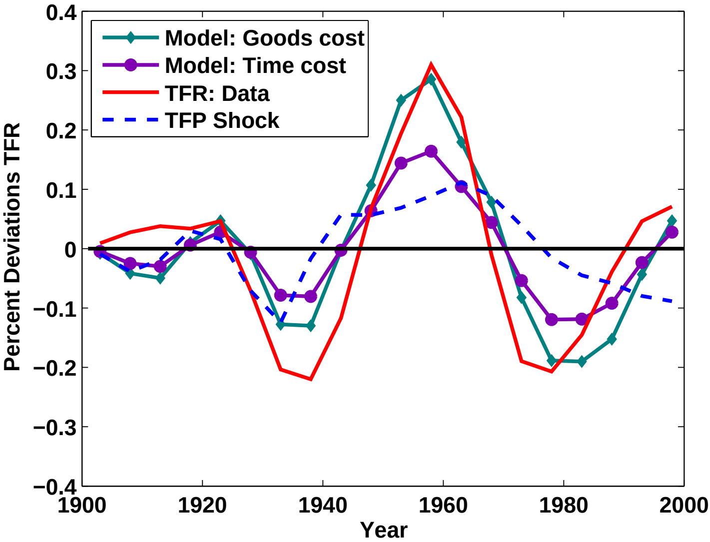
| Decade | 1910s | 1930s | 1950s | 1970s | 1990s |
|---|---|---|---|---|---|
| TFP deviat. | -0.45 | -11.2 | 8.4 | 5.4 | -6.7 |



# CTFR: All Dynasties/Cohorts



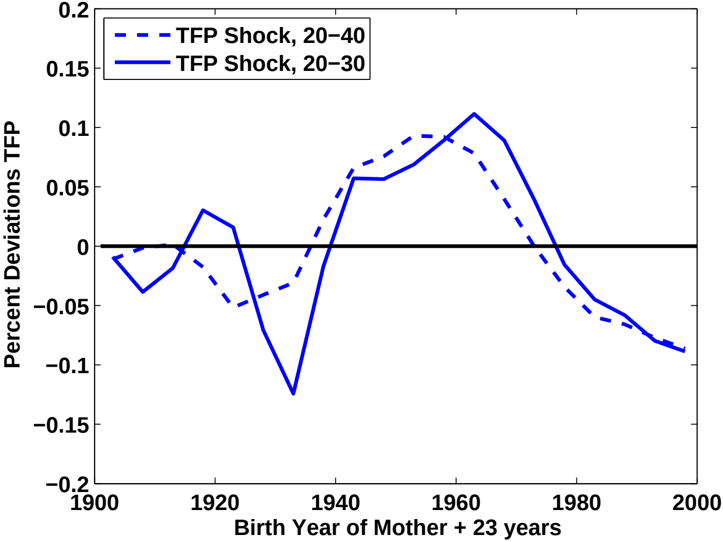
# TFR: All Dynasties/Cohorts



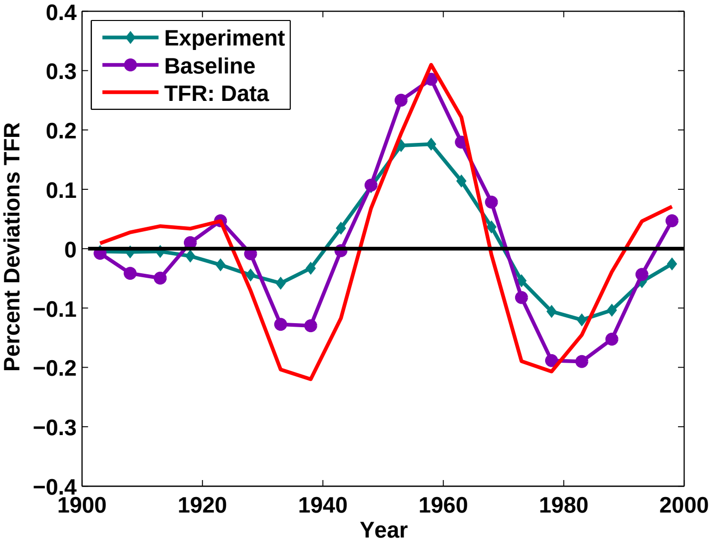
# Sensitivity

- Change shocks:
  - **Age range: 10- versus 20-year shocks**
  - Labor Productivity versus TFP
- Change state space:
  - Physical capital (with $w^2 = 0$)
  - 2 vs. 3 period work life (non-stochastic, assume bust)
- Parameter sensitivity

[INT]


TFP Shock, 20−40
TFP Shock, 20−30
Percent Deviations TFP
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
1900
1920
1940
1960
1980
2000
Birth Year of Mother + 23 years

# TFR: All Dynasties/Cohorts - 10 versus 20 year shock

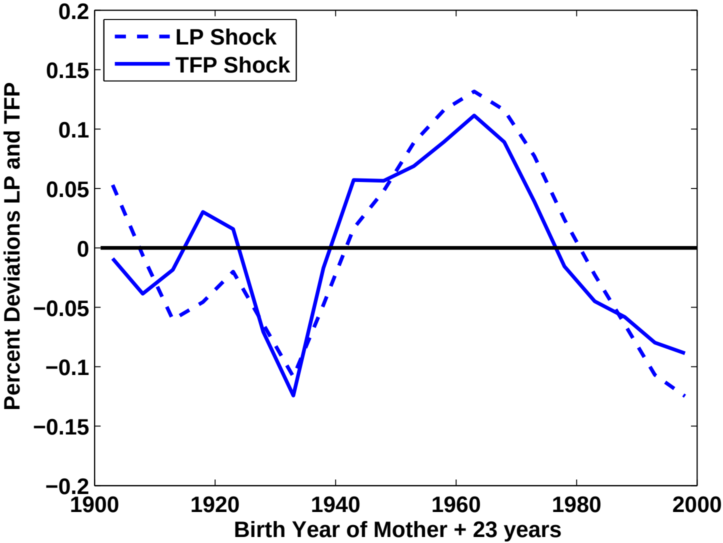


# Sensitivity

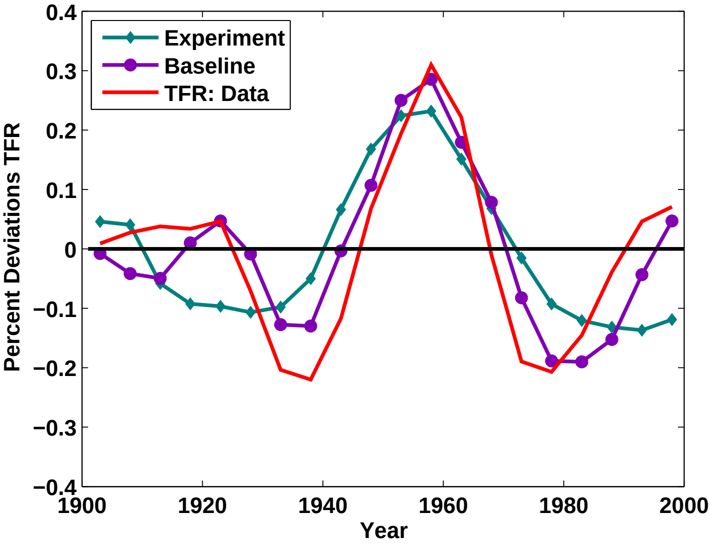
- Change shocks:
  - Age range: 10- versus 20-year shocks
  - **Labor Productivity versus TFP**
- Change state space:
  - Physical capital (with $w^2 = 0$)
  - 2 vs. 3 period work life (non-stochastic, assume bust)
- Parameter sensitivity

[INT]

# Labor Productivity versus TFP shocks



# TFR: All Dynasties/Cohorts — LP versus TFP



# Sensitivity

- Change shocks:
  - Age range: 10- versus 20-year shocks
  - Labor Productivity versus TFP
- Change state space:
  - **Physical capital (with $w^2 = 0$)**
  - 2 vs. 3 period work life (non-stochastic, assume bust)
- Parameter sensitivity

[INT]

# Model with $K$ ($w^2 = 0$, goods cost)

$$V(N^1, K, s) = \max_{(C, N^{1\prime}, K^\prime)} \left\{ T \frac{C^{1-\sigma}}{1-\sigma} + \beta\gamma^{1-\sigma} E\left(V(N^{1\prime}, K^\prime, s^\prime)\right) \right\}$$

subject to:

$$TC + \theta N^{1\prime} + \gamma K^\prime = sF(K, N^1) + (1-\delta)K$$

$$v(k, s) = \max_{c, n^\prime, k^\prime} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta\gamma^{1-\sigma}(n^\prime)^{1-\sigma} E\left(v(k^\prime, s^\prime)\right) \right\}$$

subject to:

$$0 \le c \le sf(k) - \gamma k^\prime n^\prime + (1-\delta)k - \theta n^\prime$$

$$\gamma k^\prime n^\prime - (1-\delta)k \ge 0$$

$N_0, K_0, s_0$ given

A 1% increase in productivity, $s$, generates:

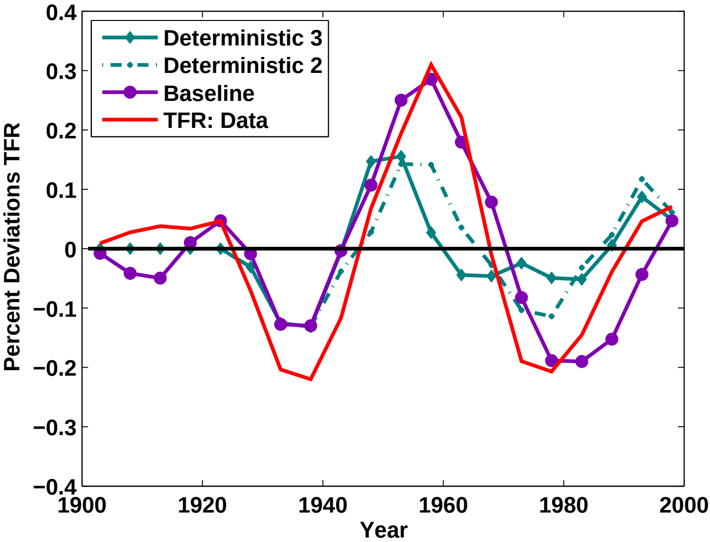
- Baseline but $w^2 = 0$: a 1% increase in fertility.
- With capital, $K$: a 0.9938% increase in fertility.

# Sensitivity

- Change shocks:
  - Age range: 10- versus 20-year shocks
  - Labor Productivity versus TFP
- Change state space:
  - Physical capital (with $w^2 = 0$)
  - **2 vs. 3 period work life (non-stochastic, assume bust)**
- Parameter sensitivity

[INT]

# TFR: All Dynasties/Cohorts — $w^3 = 0$ vs. $w^3 > 0$



# International Evidence

Issues:

$\rightarrow$ data availability: annual CBR and GDP for 17 countries,

$\rightarrow$ effects of wars: dummies or no dummies,

$\rightarrow$ detrending: OLS or HP filter.

We find:

- 1930s: Larger Baby Busts are strongly associated with larger Depressions.
- 1950s: Larger Baby Booms are (less strongly) associated with larger Baby Busts, if GDP deviation in the 1950s is taken into account.

# Summary of results

- Fertility can be either pro- or counter-cyclical,

  —even in simple models

  Increase in female labor supply

  $\Rightarrow$ opportunity cost of time more important

  $\Rightarrow$ fertility less pro-cyclical?

- No catching up without age-structure

  Capital $\rightarrow$ doesn't change result

- Interesting magnitudes of effects

  Current recession?