Barro-Becker Model of Fertility Choice

Lecture 2, Barro-Becker

Economic Policy in Development 2, Part 2

April 27, 2007

Intro

Fertility is negatively correlated with economic development. It is very high, as high as seven children per women, in developing countries; and extremely low in many rich countries, often well below the replacement rate. Fertility rates are often discussed by policy-makers, who argue, for example, that lowering fertility in poor countries could lead to increased economic growth. At the same time, many OECD governments have recently become concerned about low fertility rates in relation to the viability of pensions systems in the future. In response, several countries have instituted policies that subsidize child-rearing in an effort to increase the fertility rate.

Intro

In these notes simple models of fertility choice are described. In here we study the qualitative and quantitative properties of the becker barro and barro becker (hereafter B&B) models of fertility along with one based on the ideas of Caldwell (1978) on the demand for children as a source of old age support, see boldrin jones (B&J henceforth) for the model formalization.

Intro

The B&B model of fertility is based on the assumption that parents get direct utility from the consumption of their children, their grandchildren, etc. That is, parents have forward looking altruism, and, thus, they view their fertility decisions as extensions of their own dynastic family. This forms the basis for their demand for children in the B&B model.

Intro

In contrast, much of the literature in demography, focuses on a demand for children arising out of a need for support in old age. Various authors have implemented this idea, but, a natural candidate is to model this as a situation in which children care about the welfare of their parents, i.e., as 'reverse altruism.' That is, parents' demand for children arises because they know that children care about them, and will, because of this, help support them in old age. This is the focus of the B&J implementation of the Caldwell approach.

Of course, in reality, both of these sources of demand for children are operative.

Outline

The Barro-Becker Model of Fertility
- Budget and Feasibility Constraints
- Planner's Problem
- Equilibrium populations
- Balanced growth

Comparative statics
- Comparative statics of population growth
- Comparative statics of the CBR, surviving children
- The effects of changes in survival to adulthood: births

The US experience 1800-1990

Policy in the Barro-Becker Model

Preferences

It is assumed that parents care about three separate objects:

i) their own consumption in the period, c_t,
ii) the number of children they have, n_t, and,
iii) the average utility of their children, U_{t+1}.

Utility of the typical time t household is of the form:

$$U_t = u(c_t) + \beta g(n_t) U_{t+1}.$$

Intuition

Intuitively, it makes sense to assume:

1.) Parents like the consumption good: Utility is increasing and concave in own consumption;
2.) Parents are altruistic: Holding n_t fixed and increasing U_{t+1} increases (strictly) the utility of the parent, U_t;
3.) Parents like having children: Holding U_{t+1} fixed and increasing n_t increases (strictly) the utility of the parent.

It is also natural to assume:

4.) The increase described in 3.) is subject to diminishing returns.

Dynastic Utility

Successive substitution leads to a formulation of dynastic utility at time 0 in terms of the basic choice variables:

$$U_0 = \sum_{t=0}^{\infty} \left[\Pi_{s=1}^{t-1} g(n_s)\right] u(c_t).$$

Assume that $g(n) = n^{\eta}$, and let $N_t = \Pi_{s=1}^{t-1} n_s$, this is the total number of adult descendants alive during period t. Then $\Pi_{s=1}^{t-1} g(n_s) = g(\Pi_{s=1}^{t-1} n_s) = g(N_t)$, and so preferences for the dynasty head can be rewritten as:

$$U_0 = \sum_{t=0}^{\infty} \beta^t g(N_t) u\left[\frac{C_t}{N_t}\right],$$

where $C_t = N_t c_t$ is total consumption in period t.

Proposition: Parameter configuration 10

This leads to the following proposition.

Proposition

If either
i) $0 < \eta < 1$, $0 < \sigma < 1$ *and* $0 \leq \eta + \sigma - 1 < 1$*, or*
ii) $\sigma > 1$ *and* $\eta + \sigma - 1 \leq 0$*,*
then U *satisfies (a)-(d) above.*

Budget and Feasibility Constraints

Labor as only input:

$$\left\{ (N_t, C_t)_{t=0}^{\infty} | \forall t \geq 0, \quad \begin{matrix} C_t + \theta_t N_{bt} \leq w_t N_t \\ N_{t+1} = \pi N_t + N_{bt} \end{matrix} \right\}$$

Under this constraint set, the time 0 maximization problem has a convex constraint set and a concave objective function. Thus, the problem has unique solutions, concave value functions, etc.

Planner's Problem

$$\max_{\{C_t, N_{s,t}, N_t\}} \qquad \sum_{t=0}^{\infty} \beta^t N_t^{\eta} \left[\frac{C_t}{N_t}\right]^{1-\sigma} / (1-\sigma)$$

Subject to:

$C_t + \theta_{s,t} N_{s,t} \leq w_t N_t$, and

$N_{t+1} \leq \pi N_t + N_{s,t}$,

N_0 given,

First Order Condition

The first order condition for the stock of population in period $t+1$ in per capita variables is given by:

$$\theta_{s,t} c_t^{-\sigma} = \underbrace{\beta \left[w_{t+1} + \theta_{s,t+1}\pi \right] \gamma_{N,t}^{\eta-1} c_{t+1}^{-\sigma}}_{A} + \underbrace{\beta \frac{(\eta+\sigma-1)}{(1-\sigma)} \gamma_{N,t}^{\eta-1} c_{t+1}^{1-\sigma}}_{B} \tag{1}$$

where $\gamma_{N,t}$ is the population growth rate between t and $t+1$, or the number of children per adult in period t.

The intuition for this is as follows.

Ak Analogy

Consider the special case in which $\eta = 1 - \sigma$. Because of this, we get:

$$\left[\frac{C_{t+1}}{C_t}\right]^{\sigma} = \beta\left[\frac{w_{t+1}}{\theta_{s,t}} + \frac{\theta_{s,t+1}}{\theta_{s,t}}\pi\right] \tag{2}$$

This is the standard Euler Equation from an *Ak* model in terms of aggregate consumption, modified for the case θ_s potentially different from 1 with π corresponding to $1 - \delta$ (where δ denotes depreciation), and time varying costs and benefits, i.e., w_t and $\theta_{s,t}$.

Equilibrium

The Euler equation in (**??**) or (2) together with the feasibility constraint

$$\frac{C_t}{N_t} + \theta_{s,t}\left[\frac{N_{t+1}}{N_t} - \pi\right] = w_t. \tag{3}$$

and the initial condition N_0 completely describe the equilibrium path.

Balanced growth

Assume that $\theta_{s,t} = \gamma^t \theta_s$ and $w_t = \gamma^t w$. Let $\gamma_C = \frac{C_{t+1}}{C_t}$ be the growth rate in aggregate consumption and $\gamma_c = \frac{C_{t+1}/N_{t+1}}{C_t/N_t}$ the growth rate in per capita consumption.

The FOC can be rewritten as:

$$\frac{1}{\beta}\gamma_N^{1-\eta}\gamma^{\sigma-1} + \frac{(\eta+\sigma-1)}{(1-\sigma)}\gamma_N = \frac{\eta}{(1-\sigma)}\left[\frac{w}{\theta_s} + \pi\right] \qquad (4)$$

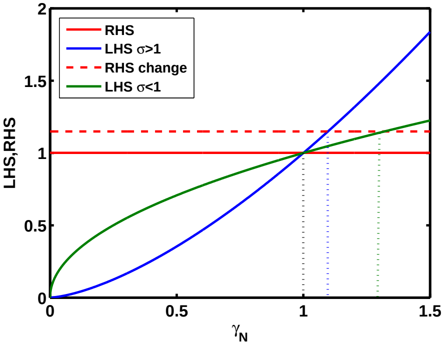
Comparative statics of population growth

Further, the above equation simplifies considerably when $1 - \sigma = \eta$. In fact,

$$\gamma_N^{\sigma} = \beta \gamma^{1-\sigma} \left[\frac{w}{\theta_s} + \pi \right]. \tag{5}$$

Comparative statics of population growth

Figure: Increase in γ_N is larger for $\sigma < 1$ than $\sigma > 1$



Comparative statics of population growth

Proposition

The following comparative statics results across BGP's hold for population growth, γ_N:

1. *An increase in γ causes γ_N to rise if $\sigma < 1$ and to fall if $\sigma > 1$;*
2. *An increase in the cost of children, θ_s, causes the population growth rate, γ_N to fall;*
3. *An increase in adult survival, π, causes γ_N to increase;*

Moreover, if $\eta = 1 - \sigma$ and $\beta\gamma^{1-\sigma}\left[\frac{w}{\theta_s} + \pi\right] > 1$ (i.e., $\gamma_N > 1$), a change in θ_s or π will have larger effects on the BGP level of γ_N if $\sigma < 1$ than if $\sigma > 1$.

Comparative statics of the CBR, surviving

In the model, we have:

$$CBR_{s,t} = \frac{N_{s,t}}{N_t + N_{s,t}} = \frac{N_{t+1} - \pi N_t}{N_t + N_{t+1} - \pi N_t} = \frac{\gamma_{Nt} - \pi}{1 + \gamma_{Nt} - \pi},$$

where $\gamma_{Nt} = \frac{N_{t+1}}{N_t}$ is the growth rate of the adult population between periods t and $t+1$.

On the BGP, CBR_s is constant and is given by:

$$CBR_s = \frac{\gamma_N - \pi}{1 + \gamma_N - \pi} = \frac{1}{1 + \frac{1}{\gamma_N - \pi}}.$$

the comparative statics results given above for γ_N for changes in γ and θ_s will also hold for CBR_s. The one exception to this concerns the effects of changes in expected life length.

Comparative statics of the CBR, surviving

Proposition

The following comparative statics results hold across BGP's for surviving children, CBR_s:

1. *An increase in γ causes CBR_s to rise if $\sigma < 1$ and fall if $\sigma > 1$;*
2. *An increase in the cost of children, θ_s, causes the fertility rate, CBR_s to fall;*
3. *If $\eta = 1 - \sigma < 0$, and $\beta\gamma^{1-\sigma}\left[\frac{w}{\theta_s} + \pi\right] > 1$ (i.e., $\gamma_N > 1$), an increase in π causes CBR_s to fall.*

Moreover, if $\eta = 1 - \sigma$ and $\beta\gamma^{1-\sigma}\left[\frac{w}{\theta_s} + \pi\right] > 1$ (i.e., $\gamma_N > 1$), a change θ_s will have larger effects on the BGP level of CBR_s if $\sigma < 1$ than if $\sigma > 1$.

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The simplest version of this is to adapt the model above as follows:

$$\max_{\{C_t, N_{b,t}, N_{s,t}, N_t\}} \qquad \sum_{t=0}^{\infty} \beta^t N_t^{\eta} \left[\frac{C_t}{N_t}\right]^{1-\sigma} / (1-\sigma)$$

subject to:

$C_t + \theta_{b,t} N_{b,t} \leq w_t N_t$, and
$N_{t+1} \leq \pi N_t + N_{s,t}$,
$N_{s,t} \leq \pi_s N_{b,t}$,
N_0 given.

where $\theta_{b,t}$ is the cost of a birth, $N_{b,t}$ is the total number of births in period t, $N_{s,t}$ is the number of children that survive to working

Effects of changes in survival to adulthood: births 23

This problem can be rewritten by eliminating $N_{b,t}$ to obtain:

$$\max_{\{C_t, N_{s,t}, N_t\}} \qquad \sum_{t=0}^{\infty} \beta^t N_t^{\eta} \left[\frac{C_t}{N_t}\right]^{1-\sigma} /(1-\sigma)$$

Subject to:

$C_t + \theta_{s,t} N_{s,t} \leq w_t N_t$, and
$N_{t+1} \leq \pi N_t + N_{s,t}$,
N_0 given,

where $\theta_{s,t} = \frac{\theta_{b,t}}{\pi_s}$ is the cost of producing a surviving child.

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This is formally equivalent to the problem analyzed above but where, now, the cost of raising a child to working age depends on the survival probability – an increase in π_s decreases $\theta_{s,t}$. Because of this equivalence, the comparative statics results given above apply. For example, an increase in π_s lowers the cost of children and hence, by the argument above increases γ_N and CBR_s. Note that this interpretation here has one additional layer of subtlety because it is γ_N and CBR_s calculated in terms of *surviving* children that increase and this does not necessarily imply that, for example, CBR calculated in terms of *births* will go up.

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To make this distinction clear we will introduce one new piece of notation

$$CBR_t = \frac{N_{b,t}}{N_t + N_{s,t}} = \frac{CBR_{s,t}}{\pi_s}.$$

Thus, even though CBR_s is increasing in π_s it need not be true that CBR is.

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Proposition

The following comparative statics results hold across BGP's for total births, CBR:

1. *An increase in γ causes CBR to rise if $\sigma < 1$ and fall if $\sigma > 1$;*
2. *If $\eta = 1 - \sigma < 0$, and $\beta\gamma^{1-\sigma}\left[\frac{w}{\theta_s} + \pi\right] > 1$ (i.e., $\gamma_N > 1$), an increase in π causes CBR to fall.*

Moreover, in both cases, percentage changes in CBR are equal to percentage changes in CBR_s

The US experience 1800-1990

see paper

Policy in the Barro-Becker Model

- Cross-country Income Tax differences

  $\longrightarrow$ higher taxes, lower fertility

- Pensions

  $\longrightarrow$ larger social security system, lower fertility?

- Child-labour laws and compulsory schooling

  $\longrightarrow$ quntity-quality trade-off