Consumption and Savings (Continued)

Lecture 9

Topics in Macroeconomics

November 5, 2007
Recall that in the Solow model

▶ the savings rate was an exogenous constant (parameter)

▶ therefore aggregate investment was a constant fraction of output/aggregate income

But people respond to incentives

⇒ Analyze consumption-savings choice
Outline

Today

- Household’s consumption and savings decision
- Determinants of household’s savings (preferences, interest rate)
- Effect of capital gains taxes on savings behavior

Tomorrow
Multi–period model and the permanent income hypothesis

Later
We will introduce the household problem into the growth model (Ramsey model)
The Model

Endowment

- Exogenous flow of income in each period: $y_t$ and $y_{t+1}$

Market structure

- Perfect financial markets where the household can freely borrow and lend by holding assets or debt, $a_{t+1}$, at an interest rate $r$

Preferences

- Life-time utility is
  \[ V(c_t, c_{t+1}) = u(c_t) + \frac{1}{1 + \rho} u(c_{t+1}) \]
The Model

Budget constraint

- The agent faces two period-by-period constraints
  \[ c_t + a_{t+1} = y_t \]
  \[ c_{t+1} = y_{t+1} + (1 + r)a_{t+1} \]

- The intertemporal budget constraint
  \[ c_t + \frac{1}{1+r}c_{t+1} = y_t + \frac{1}{1+r}y_{t+1} \]
Household’s optimization problem

Given $y_t, y_{t+1}$ and $r$

$$\max_{c_t, c_{t+1}} u(c_t) + \beta u(c_{t+1})$$

s.t. $$c_t + \frac{1}{1+r} c_{t+1} = y_t + \frac{1}{1+r} y_{t+1}$$

Analytical solution

- **Euler equation**
  \[ u'(c_t) = \frac{1 + r}{1 + \rho} u'(c_{t+1}) \]

- **CES utility function**
  \[ u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \text{if } \sigma \neq 1 \]
  \[ = \log c \quad \text{if } \sigma = 1 \]

- **CES Euler equation**
  \[ \frac{c_{t+1}}{c_t} = \left( \frac{1 + r}{1 + \rho} \right)^{\frac{1}{\sigma}} \]
The intertemporal elasticity of substitution, IES, defined as

\[ \theta(c) = -\frac{u'(c)}{u''(c)c} \]

This is essentially a measure of the curvature of the utility functions and, therefore, of the willingness to accept swings in consumption over time.

With the CES utility function, the IES becomes

\[ \theta(c) = -\frac{u'(c)}{u''(c)c} = \frac{1}{\sigma} \]
With the CES utility function, the IES becomes:

\[ \theta(c) = -\frac{u'(c)}{u''(c)c} = \frac{1}{\sigma} \]

That is, \( \sigma \) is the inverse of \( \theta \): \( \theta = \frac{1}{\sigma} \)

Since \( \sigma \) is constant, \( \theta \) is constant and \( u(.) \) is said to be of CES type.

Note that with uncertainty \( \sigma \) characterizes the degree of risk-aversion and this type of utility functions are also known as constant-relative-risk-aversion (CRRA) utility functions (not addressed in this unit).
Intertemporal substitution and $r$

- $\theta = 1/\sigma$ determines the responsiveness of the slope of the consumption path to changes in the interest rate.
- Higher $r$ implies that optimal consumption grows faster over time.
- This does not depend on the time path of income.
- This is the intertemporal-substitution effect of a change in the interest rate: $(1 + r)$ is just the relative price of $c_t$ in terms of $c_{t+1}$.
Intertemporal substitution and $r$

- Thus intertemporal substitution is the standard substitution effect when the relative price of two commodities changes.
- This effect of an increase in $r$ tends to increase saving $a = y_t - c_t$. 
Wealth effect and $r$

- But, as usual, there is also a wealth effect
- Sign depends on whether consumer is borrowing or lending

Suppose $r$ increases

- Substitution effect $\Rightarrow a \uparrow$
- Income effect $\Rightarrow a$ ?
  - If initially $a = 0$, no wealth effect $\Rightarrow a \uparrow$
  - If initially $a > 0$, positive wealth effect $\Rightarrow a$ ?
  - If initially $a < 0$, negative wealth effect $\Rightarrow a \uparrow$
Substitution and Wealth effects and $r$

- If initially saving is zero, then the wealth effect is nil and the substitution effect dictates an increase in saving.

- If initially the household is borrowing, both the wealth and substitution effects go in the direction of increasing saving (or reducing borrowing).

- If the household is initially saving, then the wealth effect tends to reduce saving and the net effect is ambiguous.

Budget constraint with taxes

- The agent faces two period-by-period constraints

\[ c_t + a_{t+1} = y_t \]
\[ c_{t+1} = y_{t+1} + (1 + r(1 - \tau))a_{t+1} \]

- The intertemporal budget constraint

\[ c_t + \frac{1}{1+r(1-\tau)}c_{t+1} = y_t + \frac{1}{1+r(1-\tau)}y_{t+1} \]

Note: preferences unchanged → optimal choice may change because the budget set and relative price of consumption in \( t \) versus \( t + 1 \) changes.
Household’s optimization problem with taxes

Given $y_t, y_{t+1}, r$ and $\tau$

$$\max_{c_t, c_{t+1}} u(c_t) + \beta u(c_{t+1})$$

subject to

$$c_t + \frac{1}{1 + (1 - \tau)r} c_{t+1} = y_t + \frac{1}{1 + (1 - \tau)r} y_{t+1}$$
Euler equation with capital-gains tax

\[ \frac{c_{t+1}}{c_t} = \left( \frac{1 + r(1 - \tau)}{1 + \rho} \right)^{\frac{1}{\sigma}} \]

Let \( \hat{r} = (1 - \tau)r \) denote the after tax interest rate (effective interest rate)

→ Higher tax rate, \( \tau \), implies lower effective interest rate, \( \hat{r} \)

→ Increasing taxes affects consumers in the same way as a decrease in the interest rate (substitution of consumption from \( t + 1 \) to \( t \) & wealth effect)

Side question: In light of the Solow model for example, should we increase or decrease taxes?