Growth Theory: Review
Lecture 1

Economic Policy in Development 2, Part 2

March 9, 2009
Outline

Growth Accounting
Growth Accounting: Objective and Technical Framework
Results

From GA to Solow to Ramsey
Solow Model
Ramsey Model

Endogenous long-run growth
Ak model
Akh model
Many factors play role to determine output in a country

- Certainly, size of the labour force and capital stock do
- But also, education, government regulation, weather,...

Any theory of economic growth chooses which of these factors to emphasize as

- sources of GDP growth within countries
- explanation for differences in levels/growth rates across countries

Growth accounting:

tool to evaluate relative importance of such factors →

Theory & Policy Implications
Technical framework

- Ignore the demand side for now
- Carefully specify the supply side
  - Inputs: capital, $K$, and labour, $L$
  - Output, $Y$
  - State of technology, $A$
Technology, $F$

\[ Y = F(K, AL) \]

where

- $Y$ = output
- $K$ = capital (input / factor)
- $L$ = labour (input / factor)
- $A$ = state of technology
- $H = AL$ = effective labour

Assumptions

- Marginal products positive and diminishing
- Constant returns to scale
Marginal products

- Marginal product of labour
  - $\frac{\partial F}{\partial L} = F_L > 0$ positive
  - $\frac{\partial^2 F}{\partial L^2} = F_{LL} < 0$ and diminishing

- Marginal product of capital
  - $\frac{\partial F}{\partial K} = F_K > 0$ positive
  - $\frac{\partial^2 F}{\partial K^2} = F_{KK} < 0$ and diminishing
Constant returns to scale (CRS)

\[ F(\lambda K, A\lambda L) = \lambda F(K, AL) \text{ for } \lambda > 0 \]

- Implications of CRS
  - Size (of firms) does not matter $\rightarrow$ representative firm
  - Euler’s theorem: Factor payments exhaust the output
  - See example
Various production functions

- Capital and labour: $Y = F(K, AL) = K^\alpha (AL)^{1-\alpha}$
- Physical and Human Capital and labour:
  $Y = F(K, H, AL) = K^\alpha H^\beta (AL)^{1-\alpha-\beta}$
- Capital and Land: $Y = F(K, L)$
- Capital and population/labor: $Y = F(K, N)$

For some questions it is not important which factor is accumulable and which is not, for other questions it is (see later).
A particular exercise: Steps for growth accounting

- TFP residual, $A_t$, for $K$ only production function
- TFP residual, $A_t$, across countries: $K$ only
- TFP residual, $A_t$, including human capital
- TFP residual, $A_t$, across countries: $K$ and $H$
- Decomposing growth in GDP per worker: $K$ only
- Decomposing growth in GDP per worker: $K$ and $H$
- Summary of results
- Critique
TFP residual, $A_t$, across countries: $K$ only

UK, South Korea and India

TFP
(Accounting for Physical Capital Only)
TFP residual, $A_t$, across countries: $K$ and $H$

UK, South Korea and India

TFP
(Accounting for Physical and Human Capital)

Economic Policy in Development 2, Part 2
Summary of Results

- TFP in the UK > TFP Korea AND India
  - For given inputs output in the UK is higher than in Korea and India

- When accounting for higher educational attainment, differences in TFP are smaller
  - Adding $H$ as input shows that India is not that much less productive than the UK; educational attainment (on average) is lower
UK, Korea and India

<table>
<thead>
<tr>
<th>K and H</th>
<th>UK</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Explained by K</td>
<td>Explained by H</td>
</tr>
<tr>
<td>1965-1970</td>
<td>82%</td>
<td>11%</td>
</tr>
<tr>
<td>1970-1975</td>
<td>98%</td>
<td>21%</td>
</tr>
<tr>
<td>1975-1980</td>
<td>55%</td>
<td>13%</td>
</tr>
<tr>
<td>1980-1985</td>
<td>24%</td>
<td>11%</td>
</tr>
<tr>
<td>1985-1990</td>
<td>36%</td>
<td>6%</td>
</tr>
<tr>
<td>Average</td>
<td>59%</td>
<td>12%</td>
</tr>
</tbody>
</table>
Summary of Results

- GDP growth accounting:
  
  increase in human capital (average years of education) accounts for a major part of growth in India

- Hence, omitting human capital in growth accounting can lead to erroneous conclusions
Critique

- Interpretation of TFP?
  - Technological change?
  - Deregulation?
  - Regulation??
  - Why did trend change?

- Other factors
  - Human capital? → Done
  - Capital-skill complementarities?
  - Quality of capital?
In growth accounting
- link of inputs in period $t$ to output in period $t$
- no link of inputs or output across periods ($t$ versus $t + 1$)

Solow model links
- population/labor force, productivity and, in particular, capital stock in year $t$
  - to
- labor force, productivity and capital stock in year $t + 1$

- Solow (1956), Solow (1957) and Solow (1960)
Solow Model

- Production function: same as before
- Exogenously given savings rate $s$
- Law of motion for physical capital
Consider $K_{t+1}$ as a function of $K_t$:

\[
K_{t+1} = (1 - \delta)K_t + l_t
\]

\[
K_{t+1} = (1 - \delta)K_t + sY_t
\]

\[
K_{t+1} = (1 - \delta)K_t + sF(K_t, \bar{A}\bar{L})
\]

Since marginal product of $K$ positive,
$\rightarrow$ law of motion: increasing function

Since marginal product of $K$ diminishing
$\rightarrow$ law of motion: concave function
Solow’s law of motion

\[ K_{t+1} = (1 - \delta) K_t + s F(K_t, AL) \]

- \( K_{t+1} = K_t \) (45 degree line)
- \( K_{t+1} = (1 - \delta) K_t + s F(K_t, AL) \)
Solow: Balanced growth: $n \neq 0$ and $g \neq 0$

- Evolution of technology:
  \[ A_{t+1} = (1 + g)A_t, \]

- Evolution of population (labour force$^*$):
  \[ L_{t+1} = (1 + n)L_t \]

- Law of motion of aggregate capital
  \[ K_{t+1} = (1 - \delta)K_t + sF(K_t, A_tL_t) \]
  \[ K_{t+1} = (1 - \delta)K_t + sK_t^\alpha(A_tL_t)^{1-\alpha} \]
  \[ (1 + g)(1 + n)\hat{k}_{t+1} = (1 - \delta)\hat{k}_t + s\hat{k}_t^\alpha \]
Solow: Balanced Growth

Law of motion into capital per unit of effective labour

\[ \hat{k}_{t+1} = \frac{1}{(1+g)(1+n)} \left[ (1 - \delta)\hat{k}_t + s\hat{k}_t^\alpha \right] \]

Short-run growth rate decreases when \( n \) increases.

This can be solved for \( \hat{k}^* \), the value for which capital per unit of effective labour does not change anymore, i.e.

\[ \hat{k}_t = \hat{k}_{t+1} = \hat{k}^* \]

\[ \hat{k}^* = \left( \frac{s}{g+n+ng+\delta} \right)^{\frac{1}{1-\alpha}} \]

Higher population growth implies lower level of capital stock per unit of effective labor in the long run, but long-run growth rate of per capita variables unaffected.
Solow Model

- Unless there is exogenous technological change \( (A_{t+1} = (1 + g)A_t, g > 0) \), the economy converges to a steady state in per capita variables.

- No long-run growth except due to technological change.

- Golden Rule steady state level of consumption requires \( s = \alpha \)

Investments result from choices: the Solow model has nothing to say about savings/investment decisions → Ramsey model
Ramsey Model

- Households choose how much to save and how much to consume → dynamics result from this choice.

\[
\max_{(a_{t+1}, c_t)_{t=0}^\infty} \sum_{t=0}^\infty \beta^t u(c_t) \\
\text{s.t. } c_t + a_{t+1} = w_t + (1 + r) a_t, \text{ for all } t = 0, 1, 2, \ldots
\]

- Euler equation: In general,

\[
u'(c_t) = \beta(1 + r_{t+1})u'(c_{t+1})
\]
Ramsey Model

- Production function/firms: same as Solow model

- Still, no fertility choice, i.e. population growth exogenous.

In terms of population growth rates, Ramsey model gives same conclusions as Solow model.
Ramsey Model

- Unless there is exogenous technological change \((A_{t+1} = (1 + g)A_t, \ g > 0)\), the economy converges to a steady state in per capita variables \((g = 0)\).
- No long-run growth except due to technological change.
- Modified Golden Rule steady state level of consumption and capital are smaller than Golden Rule. This is due to impatience of households.
- If government consumption is financed with taxes on capital gains, HH save less and the steady state \((c^{*\tau_k})\) and \((k^{*\tau_k})\) are lower than with LS taxes.

How does long-run growth occur endogenously?
Ak Model

- Take Ramsey Model but change 1 assumption: $\alpha = 1$

- Thus the production function becomes: $f(k) = Ak$ (or $F(K, L) = AK$, i.e. labor does not enter into the production function)

- That is, $f'(k) = A > 0 \rightarrow$ used in Euler equation

- That is, $f''(k) = 0$. There are no more diminishing marginal returns to capital. It is constant returns to capital alone!

How does long-run growth occur endogenously?
Ak law of motion
Ak Model: Theorem

Consider the social planner’s problem with linear technology $f(k) = Ak$ and CEIS preferences. Suppose $(\beta, \sigma, A, \delta)$ satisfy

$$\beta(A + 1 - \delta) > 1 > \beta^{\frac{1}{\sigma}}(A + 1 - \delta)^{\frac{1}{\sigma} - 1}$$

Then the economy exhibits a balanced growth path where capital, output and consumption all grow at a constant rate given by

$$\frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = \frac{c_{t+1}}{c_t} = 1 + \gamma = \left[\beta(A + 1 - \delta)\right]^{1/\sigma}$$

The growth rate is increasing in $A$ and $\beta$ and it is decreasing in $\delta$ and $\sigma$. 
Setup: Production function

Production function with human capital

\[ Y_t = F(K_t, H_t) = F(K_t, h_tL_t) \]

- \( F \): Neoclassical production function (Lecture 3):
  - A1: Constant returns to scale (CRS)
    \[ F(\lambda K, \lambda H) = \lambda F(K, H) \]
  - A2: Marginal products positive and diminishing
    \[ F_K > 0, F_H > 0, F_{KK} < 0, F_{HH} < 0 \]
  - Use CRS, write \( F \) in per capita terms \( \frac{F(K,H)}{L} = F(k, h) \)
  - For example, \( f(k, h) = Ak^\alpha h^{1-\alpha} \) with \( \alpha < 1 \)

- \( H_t = h_tL_t \) is effective labor.
Social Planner’s problem

With the usual utility function, Social planner’s problem can be written as

\[
\begin{align*}
\text{max} & \quad (k_{t+1}, c_t)_{t=0}^\infty \\
\text{s.t.} & \quad c_t + k_{t+1} + h_{t+1} = F(k_t, h_t) + (1 - \delta)k_t + (1 - \delta)h_t, \\
& \quad \text{for all } t = 0, 1, 2, \ldots \\
& \quad k_0, h_0 > 0 \text{ given}
\end{align*}
\]
Balanced growth path theorem for Akh model

Akh Model: Theorem
Consider the social planner’s problem with linear technology $f(\hat{k}) = \hat{A}\hat{k}$ and CEIS preferences. Suppose $(\beta, \sigma, A, \delta, \alpha)$ satisfy

$$\beta(\hat{A} + 1 - \delta) > 1 > \beta\frac{1}{\sigma}(\hat{A} + 1 - \delta)^{\frac{1}{\sigma} - 1}$$

$$\beta(A(1 - \alpha)^{1-\alpha} \alpha^\alpha + 1 - \delta) > 1 > \beta\frac{1}{\sigma}(A(1 - \alpha)^{1-\alpha} \alpha^\alpha + 1 - \delta)^{\frac{1}{\sigma} - 1}$$

and suppose $h_0 = \frac{1-\alpha}{\alpha} k_0$. Then the economy exhibits a balanced growth path where capital, output and consumption all grow at a constant rate given by

$$\frac{\hat{k}_{t+1}}{\hat{k}_t} = \frac{k_{t+1}}{k_t} = \frac{h_{t+1}}{h_t} = \frac{y_{t+1}}{y_t} = \frac{c_{t+1}}{c_t} = 1 + \gamma = \left[\beta(\hat{A} + 1 - \delta)\right]^{1/\sigma}$$
Concluding remarks

- This model exhibits long run growth, even though some form of labor is taken into account.

- This is because the QUALITY of labor is taken into account.

- This quality, human capital, is accumulable.

- Therefore the diminishing marginal returns don’t kick in. Both factors grow simultaneously and therefore allow the economy to grow FOREVER.
Think about other Interpretations

1. Capital and land enter the production function,
   \[ Y = F(K, \mathcal{L}) \]
   \( \rightarrow \) land is not accumulable, it is fixed
   \( \rightarrow \) decreasing marginal products kick in \( \rightarrow \) steady state
   \( \rightarrow \) land is accumulable, e.g. US expansion East to West
   \( \rightarrow \) economy expands

2. Capital and population/labor enter the production function,
   \[ Y = F(K, L) \]
   \( \rightarrow \) labor is not accumulable, it is fixed
   \( \rightarrow \) decreasing marginal products kick in \( \rightarrow \) steady state
   \[ Y = F(K, N) \]
   \( \rightarrow \) population is accumulable
   \( \rightarrow \) endogenous fertility models \( \rightarrow \) economy expands