Closing the Model

- We have seen the optimal behavior of the representative consumer and the representative firm
- We now want to impose consistent behavior
- We will study a closed economy: markets are restricted to a single country
- We will impose market clearing in the country: resources produced must be consumed by economic agents in the model
- Other market clearing condition: labour inputs into production must be supplied by consumers
- The result will be our first model of the Macroeconomy
From Chapter 4

- The representative consumer faces a tradeoff between consuming and working (work/leisure)
- The consumer is paid labour income for hours worked, and buys goods from the firm
- The firm hires labour to produce output (consumption goods) which it sells to the consumer
- Both markets (goods and labour) need to clear
The Government Budget Constraint

\[ G = T \]

- \( G \) is the amount of government spending (these are consumption goods that also need to be produced)
- \( T \) is the total amount of taxes collected by the government (these are also consumption goods)
- The government budget constraint imposes that government spending be financed by taxes
- We will assume that \( G \) is exogenous, that is, determined outside of the model
- Fiscal policy refers to the government’s choice over spending and taxes
- Other fiscal instruments: government debt and transfers
Endogenous and Exogenous Variables

- **Exogenous variables** are determined outside the model: they are taken as given
- **Endogenous variables** are determined by the model: we have to find those

- Exogenous variables in our model:
  \[ G, z, \text{ and } K \]

- Endogenous variables in our model:
  \[ C, N^s, N^d, T, Y, \pi \text{ and } w \]
Definition: Competitive Equilibrium

A competitive equilibrium is a set of endogenous variables \((C, N^s, N^d, T, \text{ and } Y)\) and an endogenous real wage rate \((w)\) such that, given exogenous variables \((G, z, \text{ and } K)\), the following conditions are satisfied:

1. Given \((w, T \text{ and } \pi)\), the bundle \((C, N^s)\) maximizes the consumer's utility subject to the budget constraint
2. Given \((w, z, \text{ and } K)\), the labor demand \((N^d)\) maximizes profits \((\pi = Y - wN^d \text{ is paid to consumers as dividend})\)
3. The government budget constraint is satisfied \((G = T)\)
4. Markets clear:
   - Labour market: \(N^d = N^s = N\)
   - Goods market: \(C + G = Y\)
Redundance of One Market Clearing Condition

If the labour market clears, so will the goods market!

- The consumer’s budget constraint has to hold:
  \[ C = wN^s + \pi - T \]
- But dividend income is the firm’s profits:
  \[ \pi = Y - wN^d \]
- If we replace profits in the consumer’s budget constraint:
  \[ C = wN^s + Y - wN^d - T \]
- Since \( N^d = N^s = N \), these terms cancel out. And since the government budget constraint holds, \( T = G \)
- It follows that \( C = Y - G \) or \( C + G = Y \) – our goods market clearing condition
This is our good old production function

Notice that the maximum number of hours that firms can hire is $h$, where the firm produces $Y^* = zF(K, h)$

When no labour is used, nothing is produced $F(K, 0) = 0$
We can change the horizontal axis from $N$ to $\ell$

The production function can be written $zF(K, h - \ell)$

The minimum number of hours of leisure ($\ell = 0$) occurs when $N = h$, where the firm still produces $Y^*$

As before, when no labour is used ($\ell = h$), nothing is produced

The slope is now equal to $-MP_N$
We can change the vertical axis from $Y$ to $C$ since $C = Y - G$ (NOT $Y^* - G$)

The maximum amount of consumption is $Y^* - G$ (point $D$)

When nothing is produced, consumption is equal to $-G$

The shaded area is the production possibilities set

The arc $DA$ is the production possibilities frontier
The production possibilities frontier (PPF) describes what the economy can produce as a whole, in terms of production of consumption and leisure.

All points in the production possibilities set are technologically possible to produce.

The slope of the PPF \(-MP_N\) is also called the marginal rate of transformation (MRT).

\(MRT_{\ell,C}\) is the rate at which leisure can be converted into consumption goods.

So we have: \(MRT_{\ell,C} = MP_N = \text{--slope of the PPF}\)
Recall that firms optimize when $\frac{MRT_{\ell,C}}{} = \frac{MP_N}{w}$.

So if $w$ is an equilibrium wage rate, then $AD$ with slope $-w$ is tangent to the PPF.

But if $w$ is an equilibrium wage rate, then $ADB$ is the budget constraint.

Recall that consumers optimize when the budget line is tangent to the indifference curve, i.e. when $\frac{MRS_{\ell,C}}{} = w$. 

Equilibrium Profits: Firm’s Perspective

\[ \pi^* = zF(K, h - \ell^*) - w^*(h - \ell^*) \]

- \[ Y^* = zF(K, h - \ell^*) = C^* + G \]
- At point J: \[ C = A - w^*\ell^* \]
- At point D: \[ C = A - w^*h \]
- The vertical distance between these two points is \[ A - w^*\ell^* - (A - w^*h) \], which is equal to the wage bill \( w^*(h - \ell^*) \)
- Subtracting \( w^*(h - \ell^*) \) from \( Y^* \) give us profits: distance \( DH \)
Recall that at point $D$, the consumer does not work at all so consumption must equal $\pi^* - T$.

Since $\pi^* - T = \pi^* - G$, the distance $DH$ is total profits again.
Pareto Optimality

Definition
An allocation is **Pareto Optimal** if there is no way to rearrange production or to re-allocate resources so that someone is made better off without making someone else worse off

- To find efficient allocations we use a benevolent Social Planner
- The Planner wants to make the representative consumer as well off as possible
- The Planner does not face markets – it chooses quantities:
  - How many hours the consumer works ⇒ $Y = zF(K, N)$
  - $G$ is given to the government
  - $Y - G$ is given to the consumer
Point $B$ is the Pareto optimum: 
\[ MRS_{\ell,C} = MRT_{\ell,C} = MP_N \]

To the left of point $B$: 
\[ MRT_{\ell,C} < MRS_{\ell,C} \]  
Could make the consumer better off by giving him more leisure (less work) and less consumption.

To the right of point $B$: 
\[ MRT_{\ell,C} > MRS_{\ell,C} \]  
Could make the consumer better off by giving him less leisure (more work) and more consumption.
Welfare Theorems

First Welfare Theorem
The first fundamental theorem of welfare economics states that, under certain conditions, a competitive equilibrium allocation is Pareto optimal.

Second Welfare Theorem
The second fundamental theorem of welfare economics states that, under certain conditions, a Pareto optimal allocation can be decentralized as a competitive equilibrium.

How does this look on a graph?
Welfare Theorems

- The competitive equilibrium allocation \((C^*, \ell^*)\) is Pareto optimal since moving away from point \(B\) makes the consumer worse off.

- The Pareto efficient allocation can be decentralized as a competitive equilibrium under price \(w^*\), the slope of the line that is tangent to the indifference curve and the PPF.
Sources of Inefficiencies

- **Externalities**
  Whereas the Planner would take externalities into account, individuals and firms DO NOT

- **Distortionary taxes**
  For example, a proportional labour income tax – consumers and firms do not face the same price (the competitive equilibrium will have $\text{MRS}_{\ell,C} < \text{MRT}_{\ell,C}$ and is therefore not Pareto optimal)

- **Firms may have market power**
  Firms with market power do not take price as given, as they know they can influence them, which typically leads to under-production
Since the welfare theorems hold, we can either work with the CE concept or the Planner’s problem.

In general, the Planner’s problem is considerably easier to work with than the CE since we need not worry about prices.

Efficiency dictates having a point of tangency between the indifference curve and the PPF (point B).

We can always find prices to decentralize the allocation as a CE.
Increase in $G$

- With $G_2 > G_1$, the PPF shifts down from $PPF_1$ to $PPF_2$ (same vertical distance everywhere).
- Note that at each $\ell$, the slope of the PPF is the same as before.
- Since $G = T$, an increase in $G$ means an increase in taxes for consumers.
- This part is exactly like a pure (negative) income change: since goods are normal, we should expect both $C$ and $\ell$ to decrease.
Increase in $G$: Crowding Out

- Notice that $\ell_2 < \ell_1$, or $N_2 > N_1$
- Production must increase ($\Delta Y = Y_2 - Y_1 > 0$)
- Since $C_1 = Y_1 - G_1$ and $C_2 = Y_2 - G_2$, 
  $C_2 - C_1 = Y_2 - G_2 - (Y_1 - G_1)$, or $\Delta C = \Delta Y - \Delta G$
- Since $\Delta Y > 0$ we must have $\Delta C > -\Delta G$ ($AE$ vs $AD$)
- Consumption is crowded out by government spending, but not completely
Increase in $G$: Equilibrium Effect

- How can we make firms hire more labour?
- The real wage rate must fall
- As $N$ increases, the $MP_N$ decreases, so $w_2 < w_1$
- But individuals still want less $\ell$ (even if it is cheaper) because the pure income effect from higher taxes dominates

**Exercise**: Decompose the total effect into an income and a substitution effect
Increase in $G$: Predictions of the Model

In the model, an increase in government spending:

- Increases employment
- Increases output
- Decreases consumption (counter-cyclical)
- Decreases the real wage rate (counter-cyclical)

Key business cycles co-movements (from Chap. 3):

- Employment is pro-cyclical
- Consumption is pro-cyclical
- Real wage rate is ‘pro-cyclical’

We conclude that business cycles are not likely to be the result of government spending fluctuations.
Increase in $z$ and the Production Function

- With $z_2 > z_1$, the production function shifts up.
- An increase in $z$ also increases $MP_N$ at each quantity of labour input.
- Notice that the maximum amount of output ($zF(K, h)$) is higher with $z_2$ than with $z_1$.
- The amount of output produced with no labour is zero regardless of the value of $z$. 
Increase in $z$ and the PPF

- With $z_2 > z_1$, the PPF shifts outward from $BA$ to $DA$
- More consumption can be produced at any level of leisure
- Since $MP_N$ is higher, the PPF is steeper under $z_2$ than under $z_1$
- The equilibrium (or efficient) allocation moves from $F$ to $H$
Increase in $z$: Total Effect

- Consumption increases from $C_1$ to $C_2$
- Leisure could increase, decrease, or stay the same – here it remains $\ell_1$
- Production increases by the same amount as $C$ ($G$ did not change)
- The wage rate increases to $w_2$ ($N$ did not change and $MP_N$ is higher at all levels of $N$)
  → That will to be true in general
Increase in $z$: Substitution Effect

- We can construct an artificial $PPF_3$ such that it is tangent to the original indifference curve $I_1$.
- Just like we did in chap. 4, this is taking away consumption (or income) away from the consumer in order to concentrate on the substitution effect.
- Notice that $PPF_3$ implies the same wage rate as $PPF_2$ (same $MP_N$).
Increase in $z$: Substitution Effect

- With $PPF_3$, the efficient allocation is given by $D$
- The substitution effect is the move from $A$ to $D$
- Consumption increases from $A$ to $D$
- Leisure decreases from $A$ to $D$
- This is just like an increase in the wage rate in Chap. 4
Increase in $z$: Income Effect

- The income effect is the move from $D$ to $B$
- Consumption increases from $D$ to $B$
- Leisure increases from $D$ to $B$
- This is just like an increase in dividend income in Chap. 4
- Total effect:
  - Consumption: must go up
  - Leisure: uncertain
Increase in \( z \): Long-Run Predictions of the Model

In the model, increases in TFP:

- Increase output
- Increase consumption
- Increase the real wage rate
- Have an ambiguous effect on hours worked

Since WWII, we have observed:

- A rise in output
- A rise in consumption
- A rise in the real wage rate
- Roughly constant hours worked

We conclude that if the income and substitution effects roughly cancel each other out over the long run, the model is consistent with technological innovations having been key to changes in these variables.
Increase in $z$: Short-Run Predictions of the Model

In the model, an increase in TFP:

- Increases output
- Increases consumption (pro-cyclical)
- Increases the real wage rate (pro-cyclical)
- Has an ambiguous effect on hours worked (?)

Key business cycles co-movements (from Chap. 3):

- Employment is pro-cyclical
- Consumption is pro-cyclical
- Real wage rate is ‘pro-cyclical’

We conclude that fluctuations in TFP may be the primary cause of business cycles if in the short run the substitution effect dominates the income effect (Real Business Cycle Theory)