A Two-Period Model: The Consumption–Savings Decision

Chapter 8, Part 1

Topics in Macroeconomics 2

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Intertemporal Decisions

- Macroeconomics studies how key variables evolve over time.
- The simplest way to think about intertemporal decisions is in a two-period model.
- The first period is the current period (or today).
- The second period represents the future (or tomorrow).
- Key trade-off: consuming today or consuming in the future, or the consumption–savings decision.
- First: consumer behavior.
- Next: competitive equilibrium with a government and later with firms and investment.
Consumption–savings: A Dynamic Decision

- Key trade-off is between current and future consumption
- To keep things simple, we will omit the labour/leisure choice (a static decision)
- The consumer can consume all his current income today and all his future income in the future (no borrowing nor savings)
- By saving, the consumer gives up consumption today for assets, which will be exchanged for consumption in the future
- By borrowing (negative savings), the consumer can consume more today but has to sacrifice consumption tomorrow to repay the loan
Simplifying Assumptions

- Assume that there is a large number of consumers \((m)\)
- Lowercase letters denote individual variables (e.g. \(c\)) and uppercase letters denote aggregate variables (e.g. \(C\))
- Instead of the labour/leisure choice, we will assume that the consumer receives **exogenous income** in both periods:
  - \(y\) represents real income in the current period
  - \(y'\) represents real income in the future period
- Although income can differ across consumers, they all pay the same lump-sum taxes
  - \(t\) represents lump-sum taxes in the current period
  - \(t'\) represents lump-sum taxes in the future period
Budget Constraint in the Current Period

\[ c + s = y - t \]

- \( s \) is the consumer’s savings in the current period.
- The budget constraint states that consumption plus savings must equal disposable income in the current period \((y - t)\).
- If \( s > 0 \), the consumer is saving: a lender on the credit market.
- If \( s < 0 \), the consumer is dissaving: a borrower on the credit market.
- Financial assets traded are riskless bonds, issued either by the government or individuals.
Riskless Bonds

Definition
A bond is a promise to pay $1 + r$ units of the consumption good tomorrow in exchange for 1 unit of the consumption good today.

- Simplifying assumptions
  1. No default (riskless)
  2. No financial intermediation (not important)
  3. The borrowing and lending real interest rate are the same $r$

- Consumers can therefore exchange 1 unit of consumption today for $1 + r$ units of consumption tomorrow.
- So $r$ is the real interest rate on bonds.
- The relative price of future consumption in terms of current consumption is $\frac{1}{1+r}$.
Budget Constraint in the Future Period

\[ c' = y' - t' + (1 + r)s \]

- \( s \) is the consumer's savings from the previous period.
- The budget constraint states that consumption must equal disposable income in the future period \( (y' - t') \) plus gross return on savings.
- **If** \( s > 0 \), the consumer redeems his bonds on the credit market for consumption goods.
- **If** \( s < 0 \), the consumer must retire the bonds issued in the previous period out of disposable income \( (y' - t') \).
The Lifetime Budget Constraint

\[
c + \frac{c'}{1 + r} = y - t + \frac{y' - t'}{1 + r}
\]

- The budget constraint in the future period implies that

\[
s = \frac{c' - (y' - t')}{1 + r}
\]

- Replace this expression for \(s\) in the current period budget constraint to get

\[
c + \frac{c' - (y' - t')}{1 + r} = y - t
\]

- Rearranging gives the **lifetime budget constraint** above
The Lifetime Budget Constraint

\[ c + \frac{c'}{1 + r} = y - t + \frac{y' - t'}{1 + r} \]

- The LHS of the lifetime budget constraint is the present value of lifetime consumption.
- The RHS of the lifetime budget constraint is the present value of lifetime disposable income or wealth.
- Present value means: in terms of period 1 consumption goods.
- The problem, then, is to choose consumption today and tomorrow (c and c') to be as well off as possible.
- Once the consumer has chosen c and c', savings (s) can be found either from the current or future budget constraint.
The Lifetime Budget Constraint Graphically

\[ c' = (1 + r)(y - t) + y' - t' - (1 + r)c \]
\[ c' = (1 + r)we - (1 + r)c \]

where \( we \) is present-value disposable income (or wealth), i.e.

\[ we = y - t + \frac{y' - t}{1 + r} \]

- The intercept of the budget constraint is \((1 + r)we\)
- The slope of the budget constraint is \(-(1 + r)\)
The Lifetime Budget Constraint Graphically

- The vertical intercept \((1 + r)we\) is what could be consumed tomorrow if nothing were consumed today.
- The horizontal intercept \(we\) is what could be consumed today if nothing were consumed tomorrow.
- The slope of the line \(BA\) is \(-(1 + r)\).
- All points in the shaded area are feasible consumption bundles.
The Lifetime Budget Constraint Graphically

- Point $E$ is the endowment point or bundle.
- The consumer consumes his/her endowment bundle (point $E$) if savings are exactly zero ($s = 0$).
- To consume points on the line $BE$, the consumer must be a lender ($s > 0$).
- To consume points on the line $EA$, the consumer must borrow ($s < 0$).
Assumptions on Preference

1. More is always preferred to less
   - More consumption today or tomorrow is preferred to less

2. Diversity is a good thing
   - In this context this means that consumers like to smooth consumption over time

3. Current consumption and future consumption are normal goods
   - So if there is a parallel shift to the right in the budget constraint, both current and future consumption will increase
Indifference Curves

- The marginal rate of substitution of consumption today for consumption tomorrow ($MRS_{c,c'}$) is the negative of the slope of an indifference curve.

- At point $B$, the $MRS_{c,c'}$ is large because $c$ is small relative to $c'$: you are willing to trade a lot of $c'$ for an extra unit of $c$.

- At point $A$, the $MRS_{c,c'}$ is small because $c$ is large relative to $c'$: you require a lot of $c$ to give up one unit of $c'$. 

The marginal rate of substitution of consumption today for consumption tomorrow ($MRS_{c,c'}$) is the negative of the slope of an indifference curve. At point $B$, the $MRS_{c,c'}$ is large because $c$ is small relative to $c'$: you are willing to trade a lot of $c'$ for an extra unit of $c$. At point $A$, the $MRS_{c,c'}$ is small because $c$ is large relative to $c'$: you require a lot of $c$ to give up one unit of $c'$. 

Consumer Optimization for a Lender

- The consumer chooses the best bundle that is budget feasible.
- Optimization implies setting $MRS_{c,c'} = (1 + r)$.
- At point $A$: The rate at which the consumer is willing to trade current consumption for future consumption is equal to the relative price of current consumption in terms of future consumption $(1 + r)$.
- $c^* < y - t$ implies that $s > 0$. 

![Graph showing consumer optimization](graph.png)
Now the endowment bundle is point $E$.

- $c^* > y - t$ implies that $s < 0$. 

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**Chapter 8, Part 1**  
**Topics in Macroeconomics**
Increase in $y$

- The endowment point changes from $E_1$ to $E_2$
- If income in the current period increases from $y_1$ to $y_2$, the budget constraint shifts to the right by $\Delta y = y_2 - y_1$
- Similarly, $we_2 - we_1 = \Delta we = \Delta y$
- The slope of the budget constraint remains $-(1 + r)$
Increase in $y$

- Since current and future consumption are normal goods, both will increase.
- Notice that the increase in current consumption ($AF$) is smaller than the increase in current income ($AD$).
- It follows that $\Delta s = \Delta y - \Delta c > 0$.
- Conclusion: an increase in $y$
  - Increases $c$ and $c'$
  - Increases $s$
Measuring Consumption Smoothing

- The prediction of the model is that consumers want to spread an increase in income by consuming more in several periods, i.e. they smooth consumption over time.
- Recall from Chapter 3 that consumption is less volatile than income (85% as volatile).
- Furthermore, consumption is measured as purchases of durables plus non-durable goods, not the actual consumption.
- But durable (consumption) goods (e.g. a new car) are more like an investment good.
- It turns out that consumption (as measured by non-durables only) is indeed much less volatile than income, which is consistent with consumption smoothing.
Consumption Smoothing in the Data
Excess sensitivity of Consumption

- While consumption is less volatile than income in the data, it is still not smooth enough to be in line with this theory.

- Two reasons are given for this excess sensitivity of consumption:
  1. Imperfections in the credit market
     Our theory assumes that consumers can freely borrow and lend at rate $r$ to smooth consumption, which may not be a good assumption.
  2. Market prices will change if all consumers want to smooth consumption simultaneously
     We have not taken general equilibrium effects into account yet: if all consumers want to lend, the interest rate will change in a way that makes consumers want to consume!
Increase in $y'$

- If income tomorrow is expected to increase from $y'_1$ to $y'_2$, the budget constraint shifts up by $\Delta y' = y'_2 - y'_1$.
- Lifetime wealth also increases from $we_1$ to $we_2$.
- The slope of the budget constraint remains $-(1 + r)$. 
Increase in $y'$

- Since current and future consumption are normal goods, both will increase.
- Notice that current consumption increases but current income is unchanged.
- It follows that $\Delta s = -\Delta c < 0$.
- Conclusion: an increase in $y'$
  - Increases $c$ and $c'$
  - Decreases $s$
The Permanent Income Hypothesis (PIH) stipulates that the main determinant of consumption is permanent income, which is closely related to lifetime wealth.

Since a temporary change in income has a small impact on permanent income, it has a small impact on consumption.

Since permanent change in income has a large impact on permanent income, it has a large impact on consumption.

In our model, consumption increases more if income increases in both periods than if income increases in only one period — the impact on savings is much smaller.
Increase in $r$

- Decreases the horizontal intercept (present value disposable income is lower)
  \[ we_2 = y - t + \frac{y' - t'}{1 + r_2} \]
- Increases the vertical intercept
  - The value of first period income in the second period is higher than before \((1 + r_2)(y - t)\)
  - The value of second period income in the second period has not changed \((y' - t')\)
- Rotates the budget line around the endowment point, with a steeper slope of \(-(1 + r_2)\)
Increase in $r$ for a Lender: Substitution Effect

- Under $r_1$, the consumer chooses point A:
  \[ c_1 < y - t \text{ so } s > 0 \]
- Under $r_2$, the consumer chooses point B
- The substitution effect is the movement from A to D
- Since the relative price of future consumption is lower:
  - $c$ decreases
  - $c'$ increases
  - $s$ increases
Increase in $r$ for a Lender: Income Effect

- The income effect is the movement from $D$ to $B$
- Since the consumer is richer and goods are normal:
  - $c$ increases
  - $c'$ increases
  - $s$ decreases
Increase in $r$ for a Lender: Total Effect

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<tr>
<th>Effect</th>
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Increase in $r$ for a Borrower: Substitution Effect

- Under $r_1$, the consumer chooses point A:
  - $c_1 > y - t$ so $s < 0$

- Under $r_2$, the consumer chooses point B

- The substitution effect is the movement from A to D

- As before, since the relative price of future consumption is lower:
  - $c$ decreases
  - $c'$ increases
  - $s$ increases
Increase in $r$ for a Borrower: Income Effect

- The income effect is the movement from $D$ to $B$
- Since the consumer is poorer and goods are normal:
  - $c$ decreases
  - $c'$ decreases
  - $s$ increases
Increase in $r$ for a Borrower: Total Effect

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## Increase in \( r \): Aggregate Effect

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The aggregate impact depends on

- The relative size of income and substitution effects
- The number of borrowers and lenders
A Two-Period Model
Consumers
Experiments

Increase in Current Period Income
Increase in Future Income
Increase in the Real Interest Rate

The Demand for Current Consumption ($c^d$)

- An increase in current income increases current consumption less than one for one
- So the slope of $c^d$ is less than one
- The slope of $c^d$ is called the marginal propensity to consume (MPC)
- We know that the $MPC < 1$
- The $MPC$ may vary with the income level
Factors that Increase the Demand for Current Consumption

The $c^d$ curve will shift up if

- The interest rate falls and the intertemporal substitution effect dominates the income effect
- The present value of taxes decreases
- Future income increases