A Two-Period Model:
The Government and Ricardian Equivalence
Chapter 6, Part 2

Topics in Macroeconomics 2

Economics Division
University of Southampton

March and April 2010
Outline

Competitive Equilibrium

Government Budget Constraint

Definition

The Ricardian Equivalence Theorem

The Theorem

Numerical and Graphical Examples

The Ricardian Equivalence in Practice

Credit Market Imperfections and Consumption

Impact on Budget Constraints

Optimization
Budget Constraint in the Current Period

\[ G = T + B \]

- Government spending \( G \) is an exogenous variable
- Total taxes collected \( T = mt \), as each of the \( m \) consumers pay lump-sum taxes \( t \)
- The government can borrow by issuing riskless bonds \( B \)
- The current-period budget deficit \( T - G \) is financed by issuing bonds
Budget Constraint in the Future Period

\[ G' + (1 + r)B = T' \]

- Government spending \( G' \) is an exogenous variable
- Total taxes collected \( T' = mt' \)
- The government has to pay interest and principal on debt issued yesterday \((1 + r)B\)
- Note: if \( B < 0 \), the government is a net lender in the first period and collects \((1 + r)B\) from private agents in the second period
The budget constraint in the future period implies that
\[ B = \frac{T' - G'}{1 + r} \]

Replace this expression for \( B \) in the current period budget constraint to get
\[ G = T + \frac{T' - G'}{1 + r} \]

Rearranging gives the government present value budget constraint above.

The LHS is the present value of spending, which must be equal to the present value of taxes collected on the RHS.
Outline

Competitive Equilibrium
  Government Budget Constraint
    Definition

The Ricardian Equivalence Theorem
  The Theorem
  Numerical and Graphical Examples
  The Ricardian Equivalence in Practice

Credit Market Imperfections and Consumption
  Impact on Budget Constraints
    Optimization
Definition: Competitive Equilibrium

A competitive equilibrium is a set of endogenous variables for consumers \((c, c', s)\), aggregate endogenous variables \((C, C', T, T', B)\) and an endogenous real interest rate \((r)\) such that, given exogenous variables for consumers \((y, y')\) and the government \((G, G')\), the following conditions are satisfied:

1. For each consumer, given \((r, y, y', t, t')\), the bundle \((c, c')\) maximizes the consumer’s utility subject to their present value budget constraint

2. The government present value budget constraint holds

3. Markets clear:
   - Credit market: \(\sum_{i=1}^{m} s = S^p = B\)
   - Period 1 goods market: \(C + G = Y = \sum_{i=1}^{m} y\)
   - Period 2 goods market: \(C' + G' = Y' = \sum_{i=1}^{m} y'\)
Redundance of goods Market Clearing Condition 1

If the credit market clears, so will the period 1 goods market!

- Aggregate private savings is income not consumed:
  \[ S^p = Y - T - C \]

- Since the first period budget constraint of the government must hold:
  \[ B = G - T \]

- The credit market clearing condition is:
  \[ Y - T - C = G - T \]

- Rearranging, we get the first period goods market clearing condition:
  \[ C + G = Y \]
Redundance of goods Market Clearing Condition 2

If the credit market clears, so will the period 2 goods market!

► Aggregate consumption in the second period is:
\[ C' = Y' - T' + (1 + r)S^p \]

► Since second period budget constraint of the government must hold:
\[ G' + (1 + r)B = T' \]

► replace in the first equation:
\[ C' = Y' - \underbrace{(G' + (1 + r)B)}_{T'} + (1 + r)S^p, \text{ or} \]
\[ C' = Y' - G' + (1 + r)(S^p - B) \]

► Since the credit market clears \((S^p = B)\), we get the second period goods market clearing condition
Outline

Competitive Equilibrium
  Government Budget Constraint
  Definition

The Ricardian Equivalence Theorem
  The Theorem
  Numerical and Graphical Examples
  The Ricardian Equivalence in Practice

Credit Market Imperfections and Consumption
  Impact on Budget Constraints
  Optimization
The Ricardian Equivalence Theorem

Holding current and future government spending constant, a change in current taxes with an equal and opposite change in the present value of future taxes leaves the equilibrium interest rate and the consumptions of individuals unchanged.

- This theorem suggests that under certain conditions the timing of taxes does not matter.
- What matters is the present value of tax liabilities.
- **Key:** consumers realize that a tax break today is not free: taxes tomorrow will be higher, so they save the tax break.
Intuition behind the Ricardian Equivalence Theorem

- Suppose the government lowers $t$ by $\Delta t$ and increases $t'$ by $\Delta t'$
- Since the present value of government spending remains the same, so does the present value of government taxes: $T + \frac{T'}{1+r}$ remains the same, so $\Delta T' = -(1 + r)\Delta T$
- This implies that the present value budget constraint of consumers is also the same:
  $c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$
- At the same interest rate, individuals will choose the same consumption bundle $(c, c')$ as before
- The only changes are in terms of savings for individuals and the government
Since neither consumption nor income change, the period 1 budget constraint for consumers implies that $\Delta s = \Delta t$ and $\Delta S^p = \Delta T$.

Since government spending does not change, the period 1 budget constraint of the government implies that $\Delta B = \Delta T$.

This means that the credit market still clears: $\Delta S^p = \Delta B$.

Since the taxes are lower in the first period, consumers save more today as they know they will have to pay more taxes tomorrow.

Since the government has lower revenues today, it issues more debt, which is bought by consumers.
Data for the Numerical Example

<table>
<thead>
<tr>
<th>Initial ‘parameters’</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>6.00</td>
</tr>
</tbody>
</table>

- \( we = 10 - 3 + \frac{12 - 4}{1.05} = 6 + \frac{9.05}{1.05} = 14.61 \)
- \( S^p = 500(10 - 6 - 3) = 2000 - 500 \times 3 = 500 \)
- \( Y = C + G \) and the government’s PVBC holds
Change in Taxes

- Suppose the government cuts taxes from 3 to 2 units in the first period.
- Then for the government PVBC to hold, $t'$ must increase by $(1+r)$, from 4 to 5.05.
- If the interest rate remains 5%, then consumers will choose the same consumption since their PVBC is the same:
  \[ we = 10 - 2 + \frac{12-5.05}{1.05} = 14.61 \]
- \[ S^p = 500 \times (10 - 2 - 6) = 1000 = 2000 - 500 \times 2 = B: \] so $r = 5\%$ still clears the credit market.
- Private savings increased by the same amount as the tax cut, leaving national savings unchanged (at 0).
The Ricardian Equivalence Graphically

- The original endowment is at point $E_1$, and the consumer chooses bundle $A$
- When current taxes fall and future taxes increase, the endowment point moves to $E_2$
- Since the change in taxes does not affect wealth, the budget constraint does not change and bundle $A$ is still optimal
- The consumer simply saves all the current tax cut to pay higher taxes tomorrow
Outline

Competitive Equilibrium
Government Budget Constraint
Definition

The Ricardian Equivalence Theorem
The Theorem
Numerical and Graphical Examples
The Ricardian Equivalence in Practice

Credit Market Imperfections and Consumption
Impact on Budget Constraints
Optimization
The Ricardian Equivalence May Not Hold in Practice

1. All individuals may not pay the same taxes, changing the tax burden across individuals
   - We assumed that $\Delta t' = -(1 + r)\Delta t$ for all individuals
   - If some consumers received higher tax cuts than others, they would change their consumption and the interest rate would change as well
   - In practice, the government can redistribute wealth though tax policy

2. Debt may not be paid off during the lifetime of all individuals who were alive when it was issued
   - Tax cuts could benefit currently old individuals and higher taxes in the future could be paid by the current young
   - This scenario involves an intergenerational redistribution of wealth
3. Lump-sum taxes are not used in practice
   ▶ As we have seen, proportional wage taxation causes inefficiencies and changes in behaviour
   ▶ The same is true if the government taxes the return to savings

4. Credit markets may not be perfect
   ▶ If you increase taxes today and consumers cannot borrow, their consumption will go down
   ▶ The government can generally borrow at a lower rate than consumers
Outline

Competitive Equilibrium
  Government Budget Constraint
    Definition

The Ricardian Equivalence Theorem
  The Theorem
  Numerical and Graphical Examples
  The Ricardian Equivalence in Practice

Credit Market Imperfections and Consumption
  Impact on Budget Constraints
    Optimization
The Second Period Budget Constraint

Suppose that it is costly to intermediate borrowing and lending.

The lending rate \( r_1 \) is smaller than the borrowing rate \( r_2 \):
\[
 r_2 > r_1
\]

The first period budget constraint is unaffected.

The second period budget constraint now depends on whether you are a lender or a borrower in the first period:

- For lenders \( s > 0 \):
  \[
  c' = y' - t' + (1 + r_1)s
  \]
- For borrowers \( s < 0 \):
  \[
  c' = y' - t' + (1 + r_2)s
  \]
The Lifetime Budget Constraint

▶ The lifetime budget constraint of a lender:

\[ c + \frac{c'}{1 + r_1} = y - t + \frac{y' - t'}{1 + r_1} \]

▶ The lifetime budget constraint of a borrower:

\[ c + \frac{c'}{1 + r_2} = y - t + \frac{y' - t'}{1 + r_2} \]
The Lifetime Budget Constraint Graphically

- To the left of the endowment point \(E\) you are a lender and face interest rate rate \(r_1\).
- To the right of the endowment point \(E\) you are a borrower and face interest rate rate \(r_2 > r_1\).

The budget constraint is \(AEF\).

- The slope of the line \(AE\) is \(-\left(1 + r_1\right)\).
- The slope of the line \(EF\) is \(-\left(1 + r_2\right)\).
Outline

Competitive Equilibrium
  Government Budget Constraint
    Definition

The Ricardian Equivalence Theorem
  The Theorem
  Numerical and Graphical Examples
  The Ricardian Equivalence in Practice

Credit Market Imperfections and Consumption
  Impact on Budget Constraints
    Optimization

Chapter 6, Part 2
Note that many consumers could end up consuming their endowment $E_1$.

For this consumer:
- The borrowing rate is too high to make borrowing worthwhile.
- The lending rate is too low to make lending worthwhile.

Note that $1 + r_1 < MRS < 1 + r_2$.
Impact of a Change in Taxes

- Suppose the government lowers taxes in period 1 and increases taxes in period 2, with 
  \[ \Delta t' = -(1 + r_1)\Delta t \]
- The endowment point moves to \( E_2 \)
- The budget constraint becomes \( AE_2F \)
- At interest rate \( r_1 \), the consumer would like to consume a lot when young (point \( G \))
- Effectively, the government borrows for the consumer at rate \( r_1 \)