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Technical Appendix to "Sequential Exporting"

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Abstract

Here we show that our main results and empirical predictions are robust to: (i) adopting a demand function of the form $p(q) = \max\{d - q, 0\}$ (Non-negative prices), and to (ii) an arbitrary positive correlation across time or destinations (Non-negative correlation).

A-1 Non-negative Prices

Here we show as a result of forcing prices to be non-negative ($p(q) = \max\{d - q, 0\}$), optimal export quantities in $t = 1$ increase, while volumes in $t = 2$ remain unaffected. Since expected export profits also increase, there is also more entry. Intuitively, such a demand function "convexifies" the revenue function, providing implicit insurance to the risk neutral producer against the event of negative prices, inducing the producer to take more risk, producing larger volumes conditional on entry, and becoming more propense to enter. [1] Because the surviving threshold in $t = 2$ remains unchanged ($\mu > \tau$), there is also more exit. Therefore our empirical predictions 2 and 3 are if anything, strengthened. Since optimal export quantities in $t = 1$ increase, while volumes in $t = 2$ remain unaffected, predicted average second year growth is lower, but still positive as long as minimum marginal costs lie above expected willigness to pay. Hence, also our empirical prediction 1 survives.

More entry and larger volumes in $t = 1$ translate into higher expected first period operational profits, inducing more experimentation. And because expected first period operational profits are

[1] Technically, it just introduces a first order stochastically dominant (FSD) shift in first period profitability, irrespective of destinations.

larger, some firms that would have entered sequentially, now enter simultaneously, as well as some non-entrants now will rather enter (sequentially) than not. Therefore our propositions 1 and 2 obtain, and so do their implications for trade policy (proposition 3).

Thus, avoiding negative prices has no effect on the expected value of information either across periods or destinations. This is why in the main text we impose the (minor) technical restriction $\underline{d} > \frac{1}{2}E\mu$, instead of exposing the reader to the cumbersome technicalities displayed here.

Proposition 1 *First period export volumes are larger under a non-negative price restriction*

Proof. We want to show that:

$$q_1^{j*} \geq \widehat{q}_1^j$$

where:

$$\begin{aligned} q_1^{j*} &\in \arg\max_{q_1 \geq 0} E\left[\max\left\{\widetilde{d} - q_1, 0\right\} q_1 - (\widetilde{c} + \tau^j) q_1\right] \\ \widehat{q}_1^j &\in \arg\max_{q_1 \geq 0} E\left[\left(\widetilde{d} - q_1\right) q_1 - (\widetilde{c} + \tau^j) q_1\right] \end{aligned}$$

The corresponding necessary and sufficient FOCs are, under the assumption of independence between demand ($\widetilde{d}$) and supply ($\widetilde{c}$) shocks:

$$\begin{aligned} \underbrace{E\left\{-1_{\left\{d > q_1^{j*}\right\}} q_1^{j*}\right\} + E\max\left\{\widetilde{d} - q_1^{j*}, 0\right\}}_{MR|p \geq 0} &= \underbrace{E\widetilde{c} + \tau^j}_{MC} \\ \underbrace{-\widehat{q}_1^j + \left(E\widetilde{d} - \widehat{q}_1^j\right)}_{MR} &= \underbrace{E\widetilde{c} + \tau^j}_{MC} \end{aligned}$$

Noting that

$$E\left\{-1_{\{d > q\}} q\right\} = qE\left\{-1_{\{d > q\}}\right\} = -q\left[1 - K(q)\right] \geq -q, \forall q \in \left[\underline{d}, \overline{d}\right],$$

and

$$E\max\left\{\widetilde{d} - q_1, 0\right\} \geq \max\left\{E\widetilde{d} - q_1, 0\right\} = 1_{\{Ed > q_1\}}\left(E\widetilde{d} - q_1\right) \geq \left(E\widetilde{d} - q_1\right),$$

it follows that the marginal revenue is larger under the non-negative price restriction,

$$\left(MR|\, p \geq 0\right)(q_1) \geq MR(q_1), \forall q_1 \in \left[\underline{d}, \overline{d}\right]$$

because the marginal revenue is a non-increasing function of the quantity[2]. Since the marginal cost remains the same (MC), we have that $q_1^{j*} \geq \widehat{q}_1^j$. ■

To be able to say if there is more or less (sequential) entry, we would need to know how do expected profits compare under the non-negative price restriction relative to its absence. First, notice that:

Proposition 2 *Conditional on entry, expected first period operational profits are larger when imposing a non-negative price restriction.*

Proof. Expected first period operational profits under a non-negative price restriction are:

$$\begin{aligned}
\Psi(q_1^{j*};\tau^j) - V(\tau^j) &= \left\{\max_{q_1\geq 0} E\left[\max\left\{\widetilde{d}-q_1, 0\right\} q_1 - (\widetilde{c}+\tau^j)q_1\right]\right\} \\
&\geq \left\{\max_{q_1\geq 0}\left[\max\left\{E\widetilde{d}-q_1, 0\right\} q_1 - (E\widetilde{c}+\tau^j)q_1\right]\right\} \\
&\geq \left\{\max_{q_1\geq 0}\left[\left(E\widetilde{d}-q_1\right) q_1 - (E\widetilde{c}+\tau^j)q_1\right]\right\} = \Psi(\widehat{q}_1^j;\tau^j) - V(\tau^j)
\end{aligned}$$

Where the second inequality follows from the convexity of the max operator and Jensen's inequality, and the third from noting that $\max\left\{E\widetilde{d}-q_1, 0\right\} = 1_{\{Ed>q_1\}}\left(E\widetilde{d}-q_1\right) \geq \left(E\widetilde{d}-q_1\right), \forall q_1$. ([3]) ■

Second, it is also true that:

Corollary 3 *Operational profits under a non-negative price restriction are larger ([4])*

Proof. Notice that the definitions of $V(\tau^j)$ and of $W(\tau^B;F)$ in the main text remain unchanged by the imposition of a non-negative price-restriction. The reason being that they constitute the ex-ante evaluation of ex-post optimal entry decisions, which rule out negative prices, i.e. $\mu \geq \tau \Longrightarrow p^* \geq 0$

[2] From Leibniz's rule, we have that $\frac{\partial(MR|p\geq 0)(q_1)}{\partial q_1} = -2(1-K(q_1)) \geq -2 = \frac{\partial MR(q_1)}{\partial q_1}, \forall q_1$

[3] After some tedious algebra, it can be shown that expected first period operational profits are equal to $\Psi(q_1^{j*};\tau^j) = \mathbb{P}(d > q_1^{j*})\left(q_1^{j*}\right)^2 + V(\tau^j)$.

[4] In the case of imperfect correlation across destinations, second period optimal output of sequential entrants is based on the conditional expectation of prices. As a result, prices can also be negative and the non-negative price restriction also constraints second period optimal outputs to be larger than they would absent the restriction. But because profits are larger, the new entry cutoff would also allow for more entry, and a similar reasoning applies.

:

$$\begin{aligned} V(\tau^j) = \int_{\tau^j}^{\overline{\mu}} \left(\frac{\mu^j-\tau^j}{2}\right)^2 dG(\mu) &= E\left[1_{\{\mu^j>\tau^j\}}\left(\frac{\mu^j-\tau^j}{2}\right)^2\right] \\ &= \Pr(\mu^j>\tau^j)E\left[\left(\frac{\mu^j-\tau^j}{2}\right)^2 \middle| \mu^j>\tau^j\right]; \\ W(\tau^B;F) &= \int_{\tau^B+2F^{\frac{1}{2}}}^{\overline{\mu}} \left[\left(\frac{\mu-\tau^B}{2}\right)^2 - F\right] dG(\mu) \\ &= \Pr(\mu>\tau^B+2F^{\frac{1}{2}})E\left[\left(\frac{\mu-\tau^B}{2}\right)^2 - F \middle| \mu>\tau^B+2F^{\frac{1}{2}}\right]. \end{aligned}$$

Therefore, the previous corollary implies that:

$$\Psi(q_1^{j*};\tau^j) \geq \Psi(\widehat{q}_1^j;\tau^j), \forall j$$

■

As a result:

Corollary 4 *Both sequential and simultaneous entry strategies display higher profits under a non-negative price restriction. Therefore, the fixed cost entry thresholds under a non-negative price restriction, F_*^{Sq} and F_*^{Sm}, are less binding.*

Proof. Defining $\Psi(q_1^{j*};\tau^j) \equiv \Psi^*(\tau^j)$, $\Pi_*^{Sq} \equiv \Psi^*(\tau^A)+W(\tau^B;F)-F$, $\Pi_*^{Sm} \equiv \Psi^*(\tau^A)+\Psi^*(\tau^B)-2F$, the previous corollary implies:

$$\Pi_*^{Sq} \geq \Pi^{Sq} \text{ and } \Pi_*^{Sm} \geq \Pi^{Sm}$$

Since the profit function is decreasing in the sunk entry cost F, we immediately have:

$$F_*^{Sq} \geq F^{Sq}$$

The definition of F_*^{Sm} and the previous corollary imply that:

$$F_*^{Sm}+W(\tau^B;F_*^{Sm}) = \Psi^*(\tau^B) \geq \Psi(\tau^B) = F^{Sm}+W(\tau^B;F^{Sm})$$

Since $\frac{d(F+W(\tau^B;F))}{dF} = G(\tau^B+2F^{\frac{1}{2}}) \geq 0$, we immediately have that $F_*^{Sm} \geq F^{Sm}$. ■

Firms that in the absence of a non-negative price restriction did not enter, now adopt a sequential entry strategy, and some of the previous sequential entrants, now would rather enter simultaneously. Therefore:

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Corollary 5 $F_*^{Sq} > F_*^{Sm}$, *i.e. Proposition 1 survives a non-negative price restriction*

Proof.

$$F_*^{Sq} = \Psi^*(\tau^A) + W(\tau^B; F_*^{Sq}) > \Psi^*(\tau^A) \geq \Psi^*(\tau^B) > \Psi^*(\tau^B) - W(\tau^B; F_*^{Sm}) = F_*^{Sm}$$

where the weak inequality follows from the assumption that $\tau^A \leq \tau^B$, and the strict inequalities obtain because under perfect positive correlation, the option value of entering B sequentially is strictly positive, $W(\tau^B; F) > 0, \forall F$. ■

Consequently, our empirical predictions 2 (entry) and 3 (exit) prevail, and are even reinforced by the adoption of a non-negative price restriction. The next proposition shows that under an economically reasonable condition, also prediction 1 holds despite of being weakened:

Proposition 6 *Empirical prediction 1 holds if* $\underline{c} \geq Ed$.

Proof. From the FOC we obtain the following expression for q_1^{j*} :

$$q_1^{j*} = 1_{\{E\mu > \tau^j + \lambda\}} \frac{E\mu - (\tau^j + \lambda)}{2P(d > q_1^{j*})}$$

where $\Pr(d > q_1^{j*}) \equiv \left[1 - K(q_1^{j*})\right] \leq 1$, and $\lambda \equiv \Pr(d \leq q_1^{j*}) E\left[d | \, d \leq q_1^{j*}\right] \geq 0, \forall q_1^{j*} \in [\underline{d}, \overline{d}]$. We need to show that:

$$\underline{c} \geq Ed \Longrightarrow Eq_2^{j*} - q_1^{j*} \geq 0$$

Noting that $Eq_2^{j*} = E\widehat{q}_2^j = \frac{E[\mu | \mu > \tau^j] - \tau^j}{2}$, omitting the non-negativity restriction on quantities in the profit maximization problem, the above implication is equivalent to:

$$\underline{c} \geq Ed \Longrightarrow \frac{E\left[\mu | \, \mu > \tau^j\right] - \tau^j}{2} \geq \frac{E\mu - (\tau^j + \lambda)}{2\Pr(d > q_1^{j*})}$$

The proof proceeds in 3 steps.

Step 1: Simplifying the RHS of the above implication.

After cancelling common terms and rearranging, we can express the RHS as :

$$\Pr(d > q_1^{j*}) E\left[\mu | \, \mu > \tau^j\right] \geq E\mu - \Pr(d \leq q_1^{j*}) \left(E\left[d | \, d \leq q_1^{j*}\right] + \tau^j\right)$$

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by definition of λ. Since $E\mu = \Pr(d > q_1^{j*})E\left[\mu|\, d > q_1^{j*}\right] + \Pr(d \leq q_1^{j*})E\left[\mu|\, d \leq q_1^{j*}\right]$, plugging this expression into the above inequality and rearranging yields:

$$\Pr(d> q_1^{j*})\left\{E\left[\mu|\, \mu > \tau^j\right] - E\left[\mu|\, d > q_1^{j*}\right]\right\} \geq$$
$$\geq \Pr(d \leq q_1^{j*})\left\{E\left[\mu|\, d \leq q_1^{j*}\right] - E\left[d|\, d \leq q_1^{j*}\right] - \tau^j\right\}$$

Substituting in the definition of $\widetilde{\mu} = \widetilde{d} - \widetilde{c}$, and taking advantage of the assumption of independence between demand and supply shocks, we get:

$$\Pr(d> q_1^{j*})\left\{E\left[d|\, d > c + \tau^j\right] - E\left[d|\, d > q_1^{j*}\right] + Ec - E\left[c|\, c < d - \tau^j\right]\right\} \geq$$
$$\geq \Pr(d \leq q_1^{j*})\left\{-Ec - \tau^j\right\}$$

Noting that the proof of empirical prediction 1 in the online appendix implies that:

$$\Pr(d > q_1^{j*})\left\{Ec - E\left[c|\, c < d - \tau^j\right]\right\} \geq 0,$$

we can then move this term to the RHS of the inequality to obtain, after some simplifications:

$$\Pr(d>q_1^{j*})\left\{E\left[d|\, d > c + \tau^j\right] - E\left[d|\, d > q_1^{j*}\right]\right\} \geq$$
$$\geq -\left\{Ec - E\left[c|\, c < d - \tau^j\right]\right\} - \Pr(d \leq q_1^{j*})\left\{E\left[c|\, c < d - \tau^j\right] + \tau^j\right.$$

Therefore the RHS of the inequality is negative.

Step 2: The LHS of the inequality is positive if $c + \tau^j > q_1^{j*}, \forall c$.

It follows from an extension of the proof of empirical prediction 1 in the online appendix: [5]

$$\tau' \geq \tau \Longrightarrow E\left[\mu|\, \mu > \tau'\right] \geq E\left[\mu|\, \mu > \tau\right], \forall\left(\tau', \tau\right) \in (\underline{\mu}, \overline{\mu})$$

Step 3: $\underline{c} > Ed \Longrightarrow c + \tau^j > q_1^{j*}, \forall c$.

Notice that

$$c + \tau^j \geq \frac{c + \tau^j}{2\Pr(d > q_1^{j*})} \geq \frac{c + \tau^j - Ec - 2\tau^j}{2\Pr(d > q_1^{j*})} = \frac{c - Ec - \tau^j}{2\Pr(d > q_1^{j*})}$$

[5] The proof proceeds similarly to the proof of empirical prediction 1 in the online appendix: integrate by parts both expressions and subtract them to obtain

$$E\left[\mu|\, \mu > \tau'\right] - E\left[\mu|\, \mu > \tau\right] = \int_{\tau}^{\tau'} G(\mu|\, \mu > \tau)d\mu + \frac{G(\tau') - G(\tau)}{\left[1 - G(\tau')\right]\left[1 - G(\tau)\right]} \int_{\tau'}^{\overline{\mu}} \left[1 - G(\mu)\right] d\mu \geq 0$$

because $G(.)$ is a non-decreasing function.

and also that

$$\frac{Ed - Ec - \tau^j}{2\Pr(d > q_1^{j*})} = \frac{E\mu - \tau^j}{2\Pr(d > q_1^{j*})} \geq \frac{E\mu - (\tau^j + \lambda)}{2\Pr(d > q_1^{j*})} = q_1^{j*}$$

Since the inequality must be true for all realizations of c, if $\underline{c} > Ed$ it must be true that $\frac{c - Ec - \tau^j}{2\Pr(d > q_1^{j*})} > \frac{Ed - Ec - \tau^j}{2\Pr(d > q_1^{j*})}$ and therefore that $\forall c, c + \tau^j > q_1^{j*}$, completing the proof. ■

A-2 Non-negatively correlated export profitabilities

Here we show that our results generalize to the case of positive but imperfect statistical dependence between random variables μ^A and μ^B.

To keep the model symmetric, we assume distributions $G(\mu^A)$ and $G(\mu^B)$ are identical, although this is not essential. Upper-bar variables denote the counterparts to the variables in the main text under perfect correlation. For brevity, we denote $E\left[\mu^B \middle| \mu^A = u^A\right]$ by $E\left(\mu^B \middle| \mu^A\right)$, where u^A denotes a particular realization of the random variable μ^A.

A-2.1 Output choice:

Output decisions in A at all times and in B at $t = 1$ are taken in the same way as in the main text. Output choice in B at $t = 2$ takes into account the realization of μ^A. From the convexity of the max function and Jensen's inequality,

$$\int_{\underline{\mu}^A}^{\overline{\mu}^A} \left[\max_{q^B \geq 0} \int_{\underline{\mu}^B}^{\overline{\mu}^B} (\mu^B - \tau^B - q^B) q^B dG(\mu^B \middle| \mu^A) \right] dG(\mu^A) \geq \max_{q^B \geq 0} \int_{\underline{\mu}^B}^{\overline{\mu}^B} (\mu^B - \tau^B - q^B) q^B dG(\mu^B),$$

where $dG(\mu^B) = \int_{\underline{\mu}^A}^{\overline{\mu}^A} dG(\mu^B \middle| \mu^A) dG(\mu^A)$. Expected profits are larger when an optimal production decision in B is made taking into account the experience acquired in A. Hence, optimal output is $\overline{q}_2^B(\tau^B) = 1_{\{E[\mu^B|\mu^A] > \tau^B\}} \left[\frac{E(\mu^B|\mu^A) - \tau^B}{2} \right]$.

A-2.2 Value of the sequential exporting strategy:

The conditional expectation of random variable μ^B can be expressed as

$$E\left[\mu^B \middle| \mu^A\right] = E\mu^B + (u^A - E\mu^A) \underbrace{\int_{\underline{\mu}}^{\overline{\mu}} \left[-\frac{d}{du} G\left(w \middle| \mu^A = u\right) \right] \Bigg|_{u=u_0} dw}_{\equiv \varpi}, \tag{A-1}$$

where ϖ captures the statistical dependence between μ^A and μ^B.[6]

At $t = 2$ a firm enters market B if

$$\left(\frac{E\left[\mu^B\middle|\,\mu^A{=}\,u^A\right]-\tau^B}{2}\right)^2 \geq F \Leftrightarrow E\left(\mu^B\middle|\,\mu^A\right) \geq 2F^{1/2}{+}\tau^B. \tag{A-2}$$

Define $\overline{F}_2^B(u^A;\tau^B)$ as the F that solves (A-2) with equality. The firm enters market B at $t = 2$ if $F \leq \overline{F}_2^B(u^A;\tau^B)$. Plugging (A-1) in (A-2) yields

$$\overline{F}_2^B(u^A;\tau^B) = \left(\frac{E\mu^B{+}\varpi(u^A{-}E\mu^A) - \tau^B}{2}\right)^2,$$

which is strictly decreasing in τ^B. Comparing $\overline{F}_2^B(u^A;\tau^B)$ with its analog under perfect correlation $F_2^B(\tau^B)$, we have that $E\mu^A = E\mu^B$ implies $\lim\limits_{\varpi\to 1}\overline{F}_2^B(u^A;\tau^B) = F_2^B(\tau^B)$.

Expressed in $t = 0$ expected terms, entering market B at $t = 2$ yields profits,

$$\begin{aligned}\overline{W}(\tau^B;F) &\equiv E_{\mu^A}\left[\max\left\{\max_{q^B\geq 0}\left[(E\left[\mu^B\middle|\,\mu^A\right]-\tau^B{-}q^B)q^B\right]-F,0\right\}\right] \\ &= E_{\mu^A}\left\{1_{\{\mu^A>\mu^{*A}(\varpi)\}}\left[1_{\{E[\mu^B|\mu^A]>\tau^B\}}\left(\frac{E\left[\mu^B|\mu^A\right]-\tau^B}{2}\right)^2-F\right]\right\} \\ &= \int_{\mu^{*A}(\varpi)}^{\overline{\mu}}\left[\left(\frac{E\left(\mu^B|\mu^A\right)-\tau^B}{2}\right)^2-F\right]dG(\mu^A),\end{aligned}$$

where

$$\mu^{*A}(\varpi) \equiv \left(\frac{1}{\varpi}\right)(2F^{1/2}{+}\tau^B){-}\left(\frac{1-\varpi}{\varpi}\right)E\mu^B$$

is the cutoff realization of export profitability in A above which a sequential exporter enters in B at $t = 2$.

For expositional clarity, notice that if μ^A and μ^B follow a bivariate normal distribution with parameters $(E\mu, E\mu, \sigma, \sigma, \rho)$, the cutoff varies with $\varpi = \rho$ as follows:

$$\frac{d\mu^{*A}(\rho)}{d\rho}{=}\frac{E\mu^B{-}(2F^{1/2}{+}\tau^B)}{\rho^2}.$$

Thus, when $E\mu^B > (2F^{1/2} + \tau^B)$ the cutoff rises as ρ increases, implying a lower value from experimentation. This simply reflects the fact that, if $E\mu^B > (2F^{1/2} + \tau^B)$, it is optimal to enter market B already at $t = 1$. Conversely, when $E\mu^B < (2F^{1/2} + \tau^B)$ the cutoff falls as ρ rises,

[6] The proof of (A-1) can be found at the end of this appendix.

implying a higher value from experimentation. This indicates that experimentation becomes more worthwhile as the statistical dependence between μ^A and μ^B increases. Experimentation is most valuable in the case of perfect correlation assumed in the main text, when it is worth $W(\tau^B;F)$. Experimentation is least valuable when μ^A and μ^B are independent, when it has no value.[7]

Derivation of (A-1)**:** Here we show how the conditional expectation can be expressed as a function of the unconditional expectation, as in (A-1). Integrating by parts both expectations and taking the difference we obtain:

$$\begin{aligned} E\left[\mu^B \middle| \mu^A = u^A\right] - E\left[\mu^B\right] &= \int_{\underline{\mu}}^{\overline{\mu}} \left[G_B(w) - G\left(w \middle| \mu^A = u^A\right)\right] dw \\ &= \int_{\underline{\mu}}^{\overline{\mu}} \left[G\left(w \middle| \mu^A \leq \overline{\mu}\right) - G\left(w \middle| \mu^A = u^A\right)\right] dw \end{aligned}$$

Since $G_B(w) \equiv G\left(\mu^B \leq w, \mu^A \leq \overline{\mu}\right) = G\left(\mu^B \leq w \middle| \mu^A \leq \overline{\mu}\right) G_A(\mu^A \leq \overline{\mu}) = G\left(\mu^B \leq w \middle| \mu^A \leq \overline{\mu}\right)$, $\forall w \in \left[\underline{\mu}, \overline{\mu}\right]$, because $G_A(\mu^A \leq \overline{\mu}) = 1$. By definition, $G\left(w \middle| \mu^A \leq \overline{\mu}\right) = \int_{\underline{\mu}}^{\overline{\mu}} G\left(w \middle| \mu^A = u\right) dG_A(u)$, which inserted above yields:

$$\begin{aligned} E\left[\mu^B \middle| \mu^A = u^A\right] - E\left[\mu^B\right] &= \int_{\underline{\mu}}^{\overline{\mu}} \left[\int_{\underline{\mu}}^{\overline{\mu}} G\left(w \middle| \mu^A = u\right) dG_A(u) - G\left(w \middle| \mu^A = u^A\right)\right] dw \\ &= \int_{\underline{\mu}}^{\overline{\mu}} \left[\int_{\underline{\mu}}^{\overline{\mu}} G\left(w \middle| \mu^A = u\right) dG_A(u) - G\left(w \middle| \mu^A = u^A\right) \underbrace{\int_{\underline{\mu}}^{\overline{\mu}} dG_A(u)}_{=1}\right] dw \\ &= \int_{\underline{\mu}}^{\overline{\mu}} \int_{\underline{\mu}}^{\overline{\mu}} \left[G\left(w \middle| \mu^A = u\right) - G\left(w \middle| \mu^A = u^A\right)\right] dG_A(u)\, dw. \end{aligned}$$

Now assuming that $G\left(w \middle| .\right) \in C^1\left[\underline{\mu}, \overline{\mu}\right]$, by the mean-value theorem,

$$\exists u_0 \in \left[\underline{\mu}, \overline{\mu}\right] : G\left(w \middle| \mu^A = u\right) - G\left(w \middle| \mu^A = u^A\right) = (u - u^A)\left(\left[\frac{d}{du} G\left(w \middle| \mu^A = u\right)\right]\bigg|_{u=u_0}\right)$$

we obtain:

$$E\left[\mu^B \middle| \mu^A = u^A\right] - E\left[\mu^B\right] = \int_{\underline{\mu}}^{\overline{\mu}} \int_{\underline{\mu}}^{\overline{\mu}} \left[(u - u^A)\left(\left[\frac{d}{du} G\left(w \middle| \mu^A = u\right)\right]\bigg|_{u=u_0}\right)\right] dG_A(u)\, dw$$

[7]Under independence between μ^A and μ^B, entry in A conveys no information about profitability in B. Thus, if it is not worthwhile to enter market B at $t = 2$, it is not worthwhile entering at $t = 1$ either. Conversely, if it pays to enter market B at $t = 2$, it must pay to enter also at $t = 1$, to avoid forgoing profits in the first period. Thus, under independence waiting to enter B at $t = 2$ is never optimal. For a formal proof of this statement, see F.N. 4 below.

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Since the term $\left[\frac{d}{du}G\left(w\right|\mu^A= u)\right]\Big|_{u=u_0}$ is a constant, it follows that:

$$\begin{aligned} E\left[\mu^B\right|\mu^A= u^A\right] - E\left[\mu^B\right] &= \left(E\left[\mu^A\right] - u^A\right)\int_{\underline{\mu}}^{\overline{\mu}}\left[\left[\frac{d}{du}G\left(w\right|\mu^A= u)\right]\Big|_{u=u_0}\right]dw \\ &= \left(u^A - E\left[\mu^A\right]\right)\int_{\underline{\mu}}^{\overline{\mu}}\left(-\left[\frac{d}{du}G\left(w\right|\mu^A= u)\right]\Big|_{u=u_0}\right)dw \end{aligned}$$

We use Lehmann's (1966, p.1143-4) definition of regression dependence, which is in our context:

Definition 7 *μ^B is positively (negatively) regression dependent on μ^A if $G\left(\mu^B\leq w\right|\mu^A= u)$ is non-increasing (non-decreasing) in u.*

Our assumption of statistical dependence between μ^A and μ^B implies regression dependence. Thus we can sign the integrand in the last equality above. Finally by rearranging the last equality, we obtain (A-1): if μ^B and μ^A are positively associated, $\left[\frac{d}{du}G\left(w\right|\mu^A= u)\right]\Big|_{u=u_0} \leq 0$ and $\left(-\left[\frac{d}{du}G\left(w\right|\mu^A= u)\right]\Big|_{u=u_0}\right) \geq 0, \forall w$ so that $\int_{\underline{\mu}}^{\overline{\mu}}\left(-\left[\frac{d}{du}G\left(w\right|\mu^A= u)\right]\Big|_{u=u_0}\right)dw \geq 0$. Now if export profitability in A was better than expected ($u^A \geq E\left[\mu^A\right]$), expected export profitability to B increases ($E\left[\mu^B\right|\mu^A= u^A\right] \geq E\left[\mu^B\right]$).

Example: normal distribution. Consider a joint normal distribution of μ^A and μ^B. It is enough to compute[8]:

$$\int_{-\infty}^{+\infty}\left[-\frac{d}{du}G\left(w\right|\mu^A= u)\right]\Big|_{u=u_0}dw$$

where

$$G\left(w\right|\mu^A= u) = \int_{-\infty}^{w}\frac{1}{\sigma_B\sqrt{2\pi}\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{s-(E\mu^B+\rho\frac{\sigma_B}{\sigma_A}(u-E\mu^A))}{\sigma_B}\right]^2\right\}ds$$

is the conditional distribution of μ^B, such that $(\mu^B\left|\mu^A = u\right) \sim N(E\mu^B+\rho\frac{\sigma_B}{\sigma_A}(u-E\mu^A), \sigma_B^2(1-\rho^2))$. We note that[9]: (i) $dG\left(s\right|\mu^A= u)$ is a continuous function of $(s,u) \in R^2$, (ii) $\frac{d}{du}[dG\left(s\right|\mu^A= u)]$ exists and is continuous, and (iii) $\int_{-\infty}^{w}dG\left(s\right|\mu^A= u)\,ds$ is continuous. Therefore we can differentiate

[8] Although expression (A-1) is defined for random variables on bounded supports, we conjecture that it can be extended to random variables over unbounded supports as long as their c.d.f., say $G(\bullet)$, possess an absolute moment of order $\psi > 0$, i.e if and only if $|\mu|^{\psi-1}\left[1-G(\mu)+G(-\mu)\right]$ is integrable over $(-\infty,+\infty)$, (see Lemma 2 in Feller (1966, p.149).

[9] Facts (i) - (iii) are stated without proof, but since $\exp(-\frac{x^2}{2})$ is continuous, positive and bounded above by an integrable function $(\exp(-|x|+1) : \int_{\mathbb{R}}\exp(-|x|+1)dx = 2e)$, on $\mathbb{R}$, the proofs are left to the interested reader.

inside the integral:

$$\begin{aligned}\tfrac{d}{du}G\left(w|\,\mu^A\!=u\right) &= \int_{-\infty}^{w}\tfrac{d}{du}\left[dG\left(s|\,\mu^A\!=u\right)\right]ds\\ &= \int_{-\infty}^{w}\left[\tfrac{1}{\sigma_B\sqrt{2\pi}\sqrt{1-\rho^2}}\left(\tfrac{\rho\frac{\sigma_B}{\sigma_A}}{\sigma_B(1-\rho^2)}\tfrac{s-(E\mu^B+\rho\frac{\sigma_B}{\sigma_A}(u-E\mu^A))}{\sigma_B}\right)\times\right.\\ &\left.\times\exp\left\{-\tfrac{1}{2(1-\rho^2)}\left(\tfrac{s-(E\mu^B+\rho\frac{\sigma_B}{\sigma_A}(u-E\mu^A))}{\sigma_B}\right)^2\right\}\right]ds\\ &= -\rho\tfrac{\sigma_B}{\sigma_A}G\left(w|\,\mu^A\!=u\right),\end{aligned}$$

which substituted above yields:

$$\int_{-\infty}^{+\infty}\left[-\frac{d}{du}G\left(w|\,\mu^A\!=u\right)\right]\bigg|_{u=u_0}dw=\int_{-\infty}^{+\infty}\rho\frac{\sigma_B}{\sigma_A}G\left(w|\,\mu^A\!=u_0\right)dw=\rho\frac{\sigma_B}{\sigma_A}$$

and hence the well-known relationship:

$$E\left[\mu^B\middle|\,\mu^A\right]=E\left[\mu^B\right]+\rho\frac{\sigma_B}{\sigma_A}\left[\mu^A-E\left[\mu^A\right]\right] \tag{A-3}$$

which is a particular case of (A-1) where $\varpi\equiv\rho\frac{\sigma_B}{\sigma_A}$.

A-2.3 Choice of export strategy (extension of Proposition 1, main text):

As in the main text, $\overline{F}^{Sq}$ is the fixed cost that makes a firm indifferent between exporting sequentially and not exporting, whereas $\overline{F}^{Sm}$ makes a firm indifferent between simultaneous and sequential exporting strategies:

$$\overline{F}^{Sq}\colon\ \Psi(\tau^A)+\overline{W}(\tau^B;\overline{F}^{Sq})=\overline{F}^{Sq}, \tag{A-4}$$

$$\overline{F}^{Sm}\colon\ \Psi(\tau^B)-\overline{W}(\tau^B;\overline{F}^{Sm})=\overline{F}^{Sm}. \tag{A-5}$$

Since $\Psi(\tau^j)$ is monotonically decreasing in τ^j and $\tau^A\le\tau^B$, and since $\overline{W}(\tau^B;F)$ is non-negative, there is a non-degenerate interval of fixed costs where firms choose the sequential export strategy.

A-2.4 Comparing imperfect with perfectly correlated export profitabilities

Here we show that when profitabilities are non-negatively regression dependent, the option value of learning one's export profitability in market B by entering in market A first, $\overline{W}(\tau^B;F)$, is bounded by the option values in the two polar cases of i.i.d. distributions (below) and perfect positive correlation (above).

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We start with the lower bound. With i.i.d. marginal distributions of μ^A and μ^B we have $E\left(\mu^B\middle|\,\mu^A\right) = E\mu^B = E\mu$ and therefore $\varpi = 0$. Accordingly, the entry condition (A-2) becomes $E\mu \geq 2F^{1/2} + \tau^B$ so that

$$\lim_{\varpi\to 0} \overline{W}(\tau^B; F) = 1_{\{E\mu > 2F^{1/2}+\tau^B\}} \left[1_{\{E\mu>\tau^B\}} \left(\frac{E\mu - \tau^B}{2}\right)^2 - F \right].$$

But then entering market B sequentially is dominated by a simultaneous entry strategy at $t = 1$: $\lim_{\varpi\to 0} \overline{W}(\tau^B; F) < \Psi(\tau^B) - F$. The reason is that by entering at $t = 2$ the firm only sacrifices positive expected profits, $V(\tau^B)$, because under independence, export experience in A is useless in B..Hence $\lim_{\varpi\to 0} \overline{W}(\tau^B; F) = 0$, and the firm will never adopt a sequential entry strategy. Figure 1 below illustrates this case.[10]

Consider now the upper bound. Under perfect positive correlation between μ^A and μ^B, the

[10] Analytically, we only need to examine whether there are values of F such that $\Pi^{Sm} \leq \overline{\Pi}^{Sq}$ when $\varpi = 0$:

$$\Psi(\tau^A) + \Psi(\tau^B) - 2F \leq \Psi(\tau^A) + \lim_{\varpi\to 0} \overline{W}(\tau^B; F) - F$$

Cancelling terms and substituting the expression for $\lim_{\varpi\to 0} \overline{W}(\tau^B; F)$,

$$\Psi(\tau^B) - F \leq \mathbf{1}_{\{E\mu>2F^{1/2}+\tau^B\}} \left[\mathbf{1}_{\{E\mu>\tau^B\}} \left(\frac{E\mu - \tau^B}{2}\right)^2 - F \right]$$

According to the first indicator function, we must distinguish two cases: (i) if $E\mu > 2F^{1/2} + \tau^B$, the inequality reduces to $V(\tau^B) \leq 0$, which is false. Hence, there is no value of F that satisfies it. (ii) If $E\mu \leq 2F^{1/2} + \tau^B$, the inequality reduces to $\Psi(\tau^B) - F \leq 0$, meaning that the only values of F that satisfy the inequality are those for which early entry in B is not worth ($e_1^B = 0$). Since late entry in B is worth only when $\Psi(\tau^B) - V(\tau^B) \geq F$, $V(\tau^B) > 0$ and the above inequality imply that:

$$F \geq \Psi(\tau^B) > \Psi(\tau^B) - V(\tau^B) \geq F,$$

a contradiction. Therefore, there is no value of F either that satisfies the inequality. Consequently, the sequential entry strategy is never adopted.

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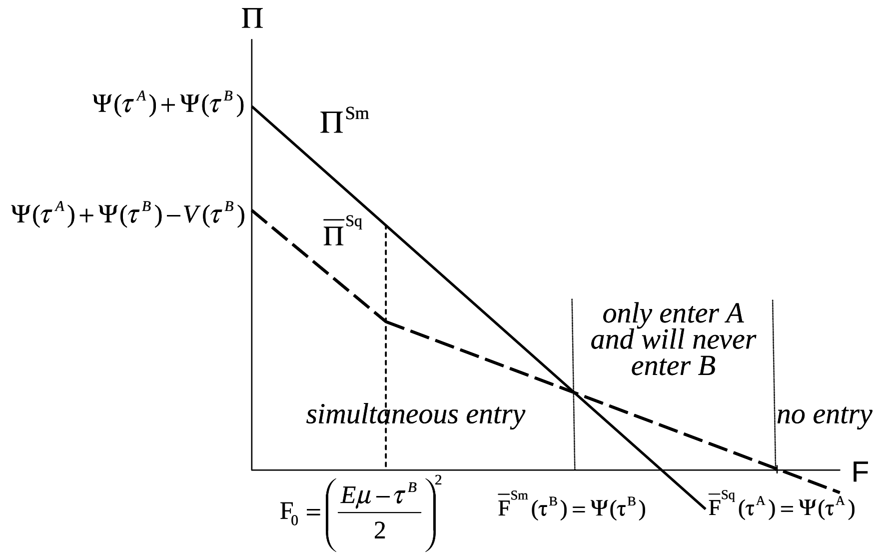

Figure 1: With independent export profitabilities ($\varpi = 0$), a firm will never enter sequentially.

term that captures the degree of statistical dependence ϖ in expression (A-1) becomes [11]:

$$\int_{\underline{\mu}}^{\overline{\mu}} \left[-\frac{d}{du} G\left(w \middle| \mu^A = u \right) \right] \Big|_{u=u_0} dw = 1.$$

Plugging this condition into expression (A-1), and since $E\mu^B = E\mu^A = E\mu$, $E\left(\mu^B \middle| \mu^A \right) = \mu^A$, we obtain that as $\varpi \to 1$:

$$\lim_{\varpi \to 1} \overline{W}(\tau^B; F) = W(\tau^B; F)$$

[11] Under perfect positive correlation between μ^A and μ^B,

$$G\left(w \middle| \mu^A = u \right) = \begin{cases} 1 & \text{if } w \geq u \\ 0 & \text{if } w < u, \end{cases}$$

which is a Heavyside step function (or unit step function) $T(w-u) = \int_{\underline{\mu}}^{u} \delta(w-s)ds$, where $\delta(w-s)$ denotes a Dirac delta function $\delta(w-s) = \begin{cases} +\infty & \text{if } w = s \\ 0 & \text{otherwise} \end{cases}$ such that $\int_{\underline{\mu}}^{\overline{\mu}} \delta(w-s)dw = 1$, $\forall s \left[\underline{\mu}, \overline{\mu} \right]$. Since $\frac{d}{du}T(w-u) = -\delta(w-u)$ we have:

$$\int_{\underline{\mu}}^{\overline{\mu}} \left[-\frac{d}{du} G\left(w \middle| \mu^A = u \right) \right] \Big|_{u=u_0} dw = \int_{\underline{\mu}}^{\overline{\mu}} \delta(w-u_0)dw = 1.$$

which is the expression in the main text. Finally, notice that:

$$\begin{aligned}
W(\tau^B;F) &= E_{\mu^B}\left[\max\left\{\max_{q^B\geq 0}(\mu^B-\tau^B-q^B)q^B-F,0\right\}\right] \\
&= E_{\mu^A}\left[E_{\mu^B|\mu^A}\left(\max\left\{\max_{q^B\geq 0}(\mu^B-\tau^B-q^B)q^B-F,0\right\}\middle|\,\mu^A\right)\right] \\
&\geq E_{\mu^A}\left[\max\left\{\max_{q^B\geq 0}E_{\mu^B|\mu^A}\left[(\mu^B-\tau^B-q^B)q^B\middle|\,\mu^A\right]-F,0\right\}\right] \\
&= E_{\mu^A}\left[\max\left\{\max_{q^B\geq 0}\left[(E\left[\mu^B\middle|\,\mu^A\right]-\tau^B-q^B)q^B\right]-F,0\right\}\right] \\
&= \overline{W}(\tau^B;F), \forall\varpi\geq 0
\end{aligned}$$

where the inequality obtains from applying twice Jensen's inequality and the convexity of the $\max\{.\}$ operator, while the third equality above follows from the law of iterated expectations, i.e. $E_{\mu^B}\left[f(\mu^B)\right]=E_{\mu^A}\left[E_{\mu^B|\mu^A}\left(f(\mu^B)\middle|\,\mu^A\right)\right]$. Therefore:

$$0\leq\overline{W}(\tau^B;F)\leq W(\tau^B;F)$$

As in the main text, those bounds on the option values correspond to sunk entry cost thresholds above which the exporter prefers to enter sequentially (F^{Sm}), as illustrated in Figure 2. [12]Hence, the region defined by Proposition 1 where it is optimal to adopt a sequential entry strategy shrinks as the statistical dependence of export profitabilities across the two destinations is reduced from

[12] Notice that in figure (in accordance with notation in the main text) $\Pi^{Sq}\equiv\overline{\Pi}^{Sq}\Big|_{\varpi=1}$ whereas, $\overline{\Pi}^{Sq}\equiv\overline{\Pi}^{Sq}\Big|_{\varpi=0}$. Also notice from the figure that $\Pi^{Sq}(F)>\overline{\Pi}^{Sq}(F),\forall F\leq\overline{F}^{Sq}\Big|_{\varpi=1}$. The only non-trivial point is to prove that $\Pi^{Sq}(0)=V(\tau^B)\geq\overline{\Pi}^{Sq}(0)=\Psi(\tau^B)-V(\tau^B)$ which follows from the application of Jensen's inequality and the convexity of the $\max\{.\}$ operator:

$$\begin{aligned}
V(\tau^B) &= E\left[\max_{q\geq 0}(\widetilde{\mu}-\tau^B-q)q\right]=E\left[\mathbf{1}_{\{\mu>\tau^B\}}\left(\frac{\mu-\tau^B}{2}\right)^2\right] \\
&\geq \max_{q\geq 0}E\left[(\widetilde{\mu}-\tau^B-q)q\right]=\mathbf{1}_{\{E\mu>\tau^B\}}\left(\frac{E\mu-\tau^B}{2}\right)^2\equiv\Psi(\tau^B)-V(\tau^B)
\end{aligned}$$

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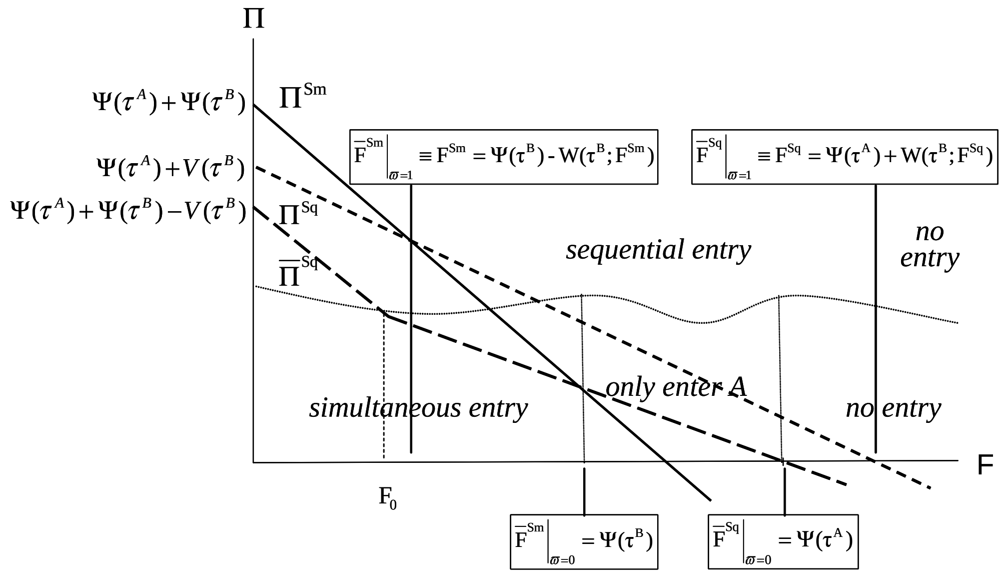

Figure 2: Bounds on sunk entry thresholds, F^{Sm} and F^{Sq}, as a function of the statistical dependence (ϖ) between export profitabilites.

perfect to no correlation:

$$\begin{aligned} F^{Sq}-F^{Sm} &\equiv \Psi(\tau^A)+W(\tau^B;F^{Sq})-\left[\Psi(\tau^B)-W(\tau^B;F^{Sm})\right] \\ &= \Psi(\tau^A)-\Psi(\tau^B)+W(\tau^B;F^{Sq})+W(\tau^B;F^{Sm}) \\ &\geq \Psi(\tau^A)-\Psi(\tau^B)+\overline{W}(\tau^B;\overline{F}^{Sq})+\overline{W}(\tau^B;\overline{F}^{Sm}) \\ &\equiv \left.\overline{F}^{Sq}-\overline{F}^{Sm}\right|_{1>\varpi>0} \\ &\geq \Psi(\tau^A)-\Psi(\tau^B) \\ &\equiv \left.\overline{F}^{Sq}-\overline{F}^{Sm}\right|_{\varpi=0} \end{aligned}$$

A-3 References

Feller, W. (1966), An Introduction to Probability Theory and Its Applications, Vol. II, Wiley Series in Probability and Mathematical Statistics, John Wiley and Sons.

Lehmann, E. L. (1966), "Some Concepts of Dependence", *The Annals of Mathematical Statistics* 37(5), 1137-1153.