Endogenous Debt Constraints in a Life-Cycle Model with an Application to Social Security*

David Andolfatto  
*Simon Fraser University  
Rimini Centre for Economic Analysis  
dandolfa@sfu.ca

Martin Gervais  
University of Southampton  
gervais@soton.ac.uk

December 17, 2007

Abstract

This paper develops a simple life-cycle model that embeds a theory of debt restrictions based on the existence of inalienable property rights à la Kehoe and Levine (1993, 2001). In our environment, net debtors have the option of defaulting on unsecured debt at the cost of being subjected to wage garnishment and/or having some or all of their future assets seized by creditors. One advantage of our framework is that it encompasses two standard versions of the life-cycle model: one with perfect capital markets and one with a non-negative net worth restriction. We study the impact of a payroll financed social security system to illustrate the role of endogenous debt constraints and compare our results to a model with exogenous debt constraints. Whereas the aggregate effects are similar under both types of constraints, the distributional consequences are found to be significantly different across debt regimes.

Journal of Economic Literature Classification Numbers: E21; E62; H55  
keywords: Life-cycle; Debt Constraints; Social Security

*We would like to thank seminar participants at the NBER Summer Institute, The SED conference in Costa Rica, The University of Western Ontario and the Richmond Fed for helpful comments and suggestions. Both authors gratefully acknowledge financial support from the Social Sciences and Humanities Research Council of Canada.
1 Introduction

This paper develops a simple life-cycle model that embeds a theory of debt restrictions based on the existence of inalienable property rights (Kehoe and Levine (1993, 2001)). In our environment, net debtors (typically young individuals with bright futures) have the option of defaulting on unsecured debt at the cost of being subjected to wage garnishment and/or having some or all of their future assets seized by creditors. One advantage of our framework is that it encompasses two standard versions of the life-cycle model: one with perfect capital markets and one with a non-negative net worth restriction.

Our environment differs in two important ways from typical models with ‘endogenous debt constraints’, such as Kehoe and Levine (1993, 2001), Kocherlakota (1996), Alvarez and Jermann (2000), and Krueger and Perri (2005). First, the income fluctuations which give rise to our debt constraint emanate purely from life-cycle considerations. Since these papers use the infinitely-lived agent abstraction, individuals need to face other kinds of fluctuations in order for the default option to be meaningful. In our framework, debt constraints arise from the simple fact that young individuals with bright futures want to smooth their life-cycle consumption. But doing so potentially entails contracting large amounts of unsecured loans. Once in their peak earning years, these individuals may find defaulting on their loans an attractive option. Since rational creditors take these incentives into account in their supply of loans, individuals may find themselves debt constrained early in their life-cycle.

Second, whereas debt constraints in our environment arise from a default option, the default option does not correspond to autarky. The default option in our framework is meant to capture in a crude way the main aspects of the bankruptcy law. In essence, bankruptcy laws restrict the extent to which creditors can garnishee wage income and seize an individual’s assets (current and future). Upon default—which does not occur in equilibrium in the model—the rights to a fraction of an individual’s current and future assets and/or wage income that can be legally garnisheed

\[ \text{In the typical framework, the penalty of defaulting is to be banned from future participation in the financial market forever. This type of setup implies not only that individual defaulters can no longer borrow, but also that they cannot lend.} \]

\[ \text{See Livshits et al. (2007) and Chatterjee et al. (2007) for detailed analyses of consumer bankruptcy.} \]
are transferred to the creditors. The permanent nature of the punishment implies that creditors will refrain from lending to individuals with a history of default. A by-product of our theory, then, is that individuals face a non-negative net worth restriction conditional on default—a familiar problem—rather than a lending constraint, which obtains when autarky is assumed following loan default. In our framework, the garnishment limits naturally translate into debt limits at the individual level: the more creditors can garnishee, the more credit is extended to individuals. In other words, harsher punishment upon default imply higher levels of debt, consistent with the work of Mateos-Planas and Seccia (2006).

Following the work of Bulow and Rogoff (1989) on sovereign debt, it is well known that the possibility of savings upon default in standard models implies that no credit will ever be extended. In other words, ‘merely denying credit is not a sufficient threat to create a loan market’ as Kehoe and Levine (2001) put it. The reason is simple: since no individual would ever pay back a loan during their last period of life, no credit will be extended to such an individual. But since it is known that default would occur in the last period of life, there is no point in extending credit the period before since individuals would default there as well, and so on. The reason why this does not occur in our economy is because of the long lasting effect of default: when an individual defaults, there is always some resources that creditors can garnishee in future periods. This feature is what makes credit markets operational from the first periods of individuals’ life.

Our framework offers a simple way of modeling debt constraints which respond endogenously to exogenous changes in the environment. As an application of our setup, we illustrate the role of endogenous debt constraints by studying the impact of introducing a payroll-tax financed social security system. We compare our results to those of the Hubbard and Judd (1987), who were among the first to examine the welfare implications of a social security system in the context of a life-cycle model with exogenous debt constraints. Because the U.S. social security systems is financed primarily through a payroll tax, the burden of finance falls disproportionately on the young and middle-aged members of the population who are approaching, or are in, their peak earning years. The provision of social security is likely to reduce the desired private saving (increase desired borrowing) among individuals in these younger cohorts. But to the extent that individuals in this subgroup are debt-constrained, a
proportional payroll tax reduces consumption expenditures dollar for dollar, and as a consequence, increases the welfare cost (reduces the benefit) of the government program. Hubbard and Judd (1987) demonstrate that these effects can be quantitatively important.

Through a parameterized version of our model, we show that the introduction of a redistributive fiscal policy such as a pay-as-you-go social security system does affect the functioning of private credit markets by tightening individuals’ debt constraints (as the supply of private loans contracts). Thus, in contrast to exogenous non-negative net-worth constraints, the consumption of debt constrained individuals may respond more than one for one in the face of a payroll tax. Our findings indicate that whereas the aggregate effect of introducing a social security system are similar whether the debt constraint is endogenous or exogenous, its distributional effects are significantly different. Rojas and Urrutia (2007) extend our analysis and show that our aggregate results are robust to the introduction of idiosyncratic uncertainty. Furthermore, they show that the transition following the elimination of social security is monotonic and that while individuals alive at the time of the announcement suffer significant losses, these losses are invariant to whether the debt constraint is endogenous or exogenous.

The rest of the paper is organized as follows. The next section lays down the economic environment and studies the consumer’s problem under endogenous debt constraints. In Section 3 we parameterize the model and investigate its behavior under different parameters governing the bankruptcy code. The impact of a payroll financed social security system under endogenous and exogenous debt constraints is the subject of Section 4. Finally, concluding remarks are offered in section 5.

2 The Economic Environment

2.1 Consumers

Consider an economy populated by overlapping generations of individuals who live for \( J \) periods, indexed by \( j = 1, 2, \ldots, J \). The population is assumed to grow at a constant rate \( n \) per period, and we denote the share of age-\( j \) individuals in the population by \( \mu_j \), which is time-invariant and satisfies \( \mu_j = (1 + n)^{-1} \mu_{j-1} \) for \( j = 2, 3, \ldots, J \) and
\[ \sum_{j=1}^{J} \mu_j = 1. \]

Individuals have preferences defined over deterministic sequences of consumption \( c_j \); these preferences are represented by a utility function \( U(c_1, c_2, \ldots, c_J) \) of the following form:

\[ U = \sum_{j=1}^{J} \beta^{j-1} u(c_j), \tag{1} \]

where \( u(\cdot) \), the per-period felicity function, has the usual properties, and \( \beta > 0 \) represents the subjective discount factor.

Individuals are endowed with one unit of time in each period of their life, which they supply inelastically. Individuals are able to transform one unit of time into \( e_j \) efficiency units of labor. The ability level of individuals within age cohort \( j \) varies according to the density function \( f_j(x) \). Note that these human capital endowments evolve deterministically and are assumed to be observable by all parties.\(^3\) Although we do not wish to diminish the role of idiosyncratic uncertainty, the idea we model here is that the most important component of life-cycle variation in earnings-capabilities is largely forecastable.

Individuals are born with zero assets and will leave the economy with zero assets; there are no bequests. Let \( w \) denote the (after-tax) price of one raw unit of labor and let \( R \) denote the gross real rate of interest. As well let \( a_j \) denote the net beginning-of-period asset position of an age-\( j \) individual. Individuals face the following sequence of budget constraints:

\[ c_j + a_{j+1} \leq w e_j + R a_j + T_j, \quad j = 1, 2, \ldots, J; \tag{2} \]

where \( a_0 = 0 \) and \( T_j \) represents a lump-sum government transfer which we later interpret as a social security payment, that is, \( T_j \) will be zero for working-age individuals and positive for retired individuals.

In the absence of any debt restrictions, the problem faced by individuals is to maximize lifetime utility (1) subject to the sequence of budget constraints (2). When \( a_j < 0 \), however, an age-\( j \) individual may find it optimal to default on his debt obligations. An individual will choose to default if the utility associated with that

\(^3\)See Andolfatto and Gervais (2006) for a life-cycle model with human capital accumulation and endogenous debt constraints.
alternative is higher than that of honoring his debt, if

\[ V_s(a_s) = \sum_{j=s}^{J} \beta^{j-s} u(c_j) \geq V_s^d, \quad s = 2, \ldots, J, \]  

(3)

where \( V_s \) and \( V_s^d \) denote the continuation payoff accruing to an age-\( s \) individual under alternative strategies of not defaulting and defaulting, respectively. We will refer to this inequality as the \textit{individual rationality constraint}.

**Default Option**  
The consequences of defaulting on a loan depend on the structure of the legal system, which limits the set of actions that creditors can take against defaulters. In the context of this model, creditors can take two actions against defaulters: garnishee their wages (current and future) and seize their assets (future). Accordingly, let \( 0 \leq \pi_w \leq 1 \) denote the fraction of wage income that a creditor is feasibly (or legally) able to garnishee and \( 0 \leq \pi_a \leq 1 \) denote the fraction of physical and/or financial assets that a creditor can feasibly seize (exclusive of the value of the social security entitlement).

We now turn to the analysis of the model. We show that given the legal structure, computing the value of the default option involves solving a familiar consumer problem with a non-negative net-worth restriction. We also show that, as one would expect, the individual rationality constraint (3) implies an age-dependent borrowing restriction.

**Consumer Problem at age \( J \)**  
The problem that an individual faces at age \( J \) depends on whether default occurred in the past. Consider first the problem faced by an individual who has never defaulted in the past and enters the last period of life with assets \( a_J \). If the asset position is positive, consuming all his income is his only option. If \( a_J < 0 \), the individual has to choose between (i) consuming his income minus the loan repayment \( (we_J + Ra_J + T_J) \) or (ii) defaulting on his loan and consuming the fraction of his income that cannot be garnisheed by creditors \( ((1 - \pi_w)we_J + T_J) \). There is thus a value of \( a_J \) under which both options are equally attractive, that is, there exists \( \bar{a}_J \) such that the individual rationality constraint (3) holds with equality:

\[ V_J(\bar{a}_J) = V_J^d \text{ when } \bar{a}_J = -\frac{\pi_w we_J}{R}. \]  

(4)
Defaulting is optimal when assets are sufficiently low ($a_J < \bar{a}_J$) for the loan repayment (principal plus interest) to be larger than the amount of resources that creditors can legally garnishee. Since this is known by creditors, the individual rationality constraint at age-$J$ effectively limits the set of asset positions that can be chosen at age $J - 1$; this constraint is given by $a_J \geq \bar{a}_J$.

Next consider the problem faced by an age-$J$ individual with a history of default. First note that when default occurred, the property rights to the defaulter’s earnings that can legally be garnisheed were transferred to creditors. Since the remainder of the defaulter's earnings is inalienable, an individual who has defaulted in the past will default again on any loan that is extended to him. Anticipating this strategy, no credit will ever be extended to an age-$J$ individual who has defaulted in the past. Note that the exact timing of the default is irrelevant for this borrowing constraint: if an individual defaulted at some point in the past, today’s earnings are being garnisheed. To summarize, the individual rationality constraint (3) implies the following borrowing constraint:

$$a_J \geq \begin{cases} -\frac{\pi w_{wB}}{R} & \text{no default history,} \\ 0 & \text{default history.} \end{cases}$$

(5)

**Generalization to Earlier Periods** To make their decisions, younger individuals need to consider the impact of defaulting on future earnings and future capital market participation in addition to the direct impact on today’s consumption. As before, we need to separate the problem according to default histories.

Consider first the problem faced by an age-$s$ individual with a history of default, entering the period with assets $a_s$. This problem is simplified by the fact that $a_s \geq 0$, so that no default decision needs to be made. To see this, note that defaulting is optimal for any $a_s < 0$, for the property rights to all the defaulter’s earnings that can legally be garnisheed were transferred to previous creditors. It follows that no creditor will extend any credit to an individual with a history of default. An individual with a history of default thus faces a non-negative net worth restriction for the rest of his life. As a result, we need not concern ourselves with the possibility that an individual may have defaulted many times in the past nor with the possibility that an individual may default again in the future.
The above argument is very helpful in analyzing the default decision of an age-$s$ individual who has never defaulted in the past. We have just established that in addition to having a fraction of earnings garnished, defaulting today eliminates an individual’s ability to borrow in the future. Hence, the continuation value of defaulting at age $s$ ($V_s^d$) is the solution to the following problem:

$$V_s^d \equiv \max \sum_{j=s}^{J} \beta^{j-s} u(c_j)$$

subject to

$$c_s + a_{s+1} \leq (1 - \pi^u) we_s + T_s$$

$$c_j + a_{j+1} \leq (1 - \pi^u) we_j + (1 - \pi^u) Ra_j + T_j, \quad j = s + 1, \ldots, J$$

$$a_{j+1} \geq 0, \quad j = s, \ldots, J - 1.$$  \tag{9}

Note that $V_s^d$ does not vary with $a_s$, but that $V_s(a_s)$ is strictly increasing in $a_s$.\footnote{This result only requires the utility function to be increasing in consumption. In general, increasing $a_s$ increases consumption in all remaining periods. However, an increase in $a_s$ may only increase current consumption if the individual is constrained. In any case, it must increase the present discounted value of utility.}

There must therefore exist an asset position $\bar{a}_s$ such that $V_s(\bar{a}_s) = V_s^d$. This asset position is the smallest amount of asset (largest amount of debt) that can be chosen at age $s - 1$. To summarize, the individual rationality constraint (3) at age $s$ implies the following borrowing constraint at age-$s - 1$:

$$a_s \geq \begin{cases} \bar{a}_s & \text{no default in history,} \\ 0 & \text{default in history.} \end{cases}$$  \tag{10}

The above analysis emphasizes that the individual rationality constraint faced by an age-$(j + 1)$ individual translates into a restriction on the set of asset positions that can be chosen at age $j$. Naturally, this borrowing constraint, or the maximum amount of debt that can be extended to an individual, depends on the same factors that affect the incentive to default (age, current asset position, and legal structure).
2.2 Technology and Feasibility

In a steady-state, the per capita stock of capital is given by:

\[ K = (1 + n)^{-1} \sum_{j=1}^{J} \mu_j \int a_{j+1}(x) df_j(x), \]

and the per capita level of hours measured in efficiency units is given by:

\[ H = \sum_{j=1}^{J} \mu_j \int e_j(x) df_j(x). \]

Output is produced according to a constant returns to scale production technology \( Y = F(K, H) \), where

\[ F(K, H) = AK^\alpha H^{1-\alpha}, \quad 0 < \alpha < 1, \quad A > 0. \]

Equilibrium factor prices are determined competitively; i.e.,

\[ \hat{w} = F_H(K, H); \]
\[ R - 1 = F_K(K, H) - \delta, \]

where \( 0 \leq \delta \leq 1 \) is the rate at which physical capital depreciates every period. Note that the after-tax wage rate is equal to \((1 - \tau)\hat{w}\). Goods-market clearing requires:

\[ C + (n + \delta)K = Y, \]

where \( C = \sum_{j=1}^{J} \mu_j \int c_j(x) df_j(x) \). Finally, government budget-balance requires that all the proceed from the payroll tax be redistributed to individuals in the form of lump-sum transfers,

\[ \tau \hat{w} H = \sum_{j=1}^{J} \mu_j T_j. \]

3 Calibration

3.1 Parameter Selection

The model’s parameters are those that describe: demographics \((n, J, \) as well as the endowment process\); preferences \((\beta, \sigma)\); technology \((A, \alpha, \delta)\); and the legal institution
Table 1: Benchmark Parameters

<table>
<thead>
<tr>
<th></th>
<th>$e_1$</th>
<th>$\bar{x}$</th>
<th>$\bar{x}$</th>
<th>$m$</th>
<th>$s$</th>
<th>$\theta$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.00</td>
<td>-5.20</td>
<td>2.40</td>
<td>-1.40</td>
<td>1.52</td>
<td>0.10</td>
<td>0.22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$A$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\pi^w$</th>
<th>$\pi^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.78</td>
<td>2.00</td>
<td>1.95</td>
<td>0.35</td>
<td>0.88</td>
<td>0.00</td>
<td>0.14</td>
</tr>
</tbody>
</table>

$(\pi_w, \pi_a)$. The parameter values chosen for the benchmark economy are reported in Table 1.

Individuals live for three periods ($J = 3$), so we take one model-period to represent twenty years of adult life. Following Hubbard and Judd (1987) the population grows at a rate of one percent per annum, so that $n = 1.01^{20} - 1$.

We assume that $e_1$ is common across individuals and normalize it to unity. This specification is meant to reflect the observation that skill levels among the young are relatively homogeneous. However, individuals face different prospects for the growth of their human capital; in particular, we assume that $e_2 = e_1 + \exp(x)$, where $x$ varies across the population according to a (truncated) normal probability density function

$$f(x) = [2(\pi s)^{1/2}]^{-1} \exp \left[-\frac{1}{2} \left(\frac{x - m}{s}\right)^2\right],$$

with $-\infty < \underline{x} \leq x \leq \bar{x} < \infty$ and where $m$ and $s$ are parameters. In our computational experiments below, we assume that the $x$ (and hence $e_2$) are elements of a 100 point grid; i.e., $e_2 \in \{\underline{x}, \ldots, \bar{x}\} = E$. For simplicity, we assume that this mid-life ability level decays at the common rate of $(1 - \theta)$, so that $e_3 = \theta e_2$ for each $e_2 \in E$.

The parameters governing the life-cycle evolution of human capital ($m, \theta$) are chosen so as to approximate typical life-cycle earnings profiles (e.g. Hansen (1993), Huggett (1996)). These profiles imply that average earnings of the 40-60 age group is close to 25% higher than that of the 20–40 age group. To match this average growth, we set $m = -1.4$. Similarly, income of the old drops sharply in these profiles, so that average income is about 10% that of the middle-aged, and hence that value was chosen for the parameter $\theta$. The standard deviation of earnings $s$ is taken directly
from PSID data. Starting with all households whose head is around 40 in 1980 (we used a 5 year age band—from 38 to 42—because of sample size), we added labor earnings until 1997 (after 1997 the survey is no longer yearly) and divided by the number of years for which income wasn’t missing—we threw out individuals who’s income was missing for 5 or more years. We then computed the standard deviation of log earnings of that sample. We set $s$ to the value found in the data, 1.52. Finally, since we have 100 individuals representing percentile of that distribution, we chose the lowest and highest values of income to be $\pm 2.5$ standard deviations away from the mean.

The subjective discount factor $\beta$ is chosen to have a capital to output ratio of 3.0 in the benchmark economy. The annual discount factor which achieves this goal is 0.988, so that $\beta = 0.9988^{20}$. The coefficient of relative risk aversion $\sigma$ is fixed at 2.0. The scaling technology parameter $A$ is chosen to make output in the benchmark economy equal to one. We fix the annual rate of depreciation of capital at 0.10, so that $\delta = 1 - (1 - 0.1)^{20}$, and set $\alpha$ to 0.35, the capital share of total income in the data, which produces an annual interest rate equal to 4.6%.

The parameter $\pi^w$ measures the degree to which creditors can recoup debt via wage garnishment, should a debtor choose to default. In the United States, Federal law stipulates that a minimum of 75% of wages or 30 times the Federal minimum hourly wage per week, whichever is higher, be exempt from garnishment. Some states (Texas, Pennsylvania, Alaska, South Dakota and Florida) prohibit wage garnishment entirely; see Fay et al. (2002). Of course, legal stipulations are one thing and their enforcement is another. In practice, it is likely very difficult for creditors to enforce debt repayment through wage garnishment, even with a court order granting them title to some of the debtor’s wage income. The primary reason for this state of affairs likely resides in the fact that indentured servitude is legally prohibited in the United States. Consequently, debtors can escape wage garnishment by reallocating time to an activity that generates an excludable consumption flow (e.g., leisure or home production), or perhaps to some underground activity. These considerations lead us to set $\pi^w = 0$.7

---

5 We use the Cross National Equivalent Files (CNEF) produced by the Department of Policy Analysis and Management at Cornell University.
6 This was not always the case; see Galenson (1981).
7 As discussed in Section 4, our results are robust to changes in this parameter value as long as
Debtors have a more difficult time protecting their physical and financial assets (relative to their human capital) from seizure by creditors. Kehoe and Levine (2001) effectively assume $\pi^a = 1.00$, so that creditors can seize all assets (in value up to the amount owed). This restriction effectively excludes defaulters from future participation in financial markets. In reality, the ability of creditors to seize property is limited. Even in the absence of personal bankruptcy laws, courts will typically limit the amount of assets that can be seized (especially if their seizure results in ‘below subsistence’ living standards). In addition, since the Bankruptcy Reform Act of 1978, generous exemptions for ‘rich debtors’ have been possible. This law allowed bankrupts to keep $7500$ in homestead equity and $4000$ in non-homestead property from creditors. In 1994, these exemption levels were doubled. Furthermore, many states have adopted even more generous exemption levels (e.g., an unlimited homestead equity feature is not uncommon). Consequently, debtors can to some extent avoid asset seizure by allocating their assets and savings to these exempt categories. In one extreme case, we could set $\pi^a = 0$, but this would result in a non-negative net worth constraint since debtors could default with impunity. While this case represents the norm commonly adopted in the literature, it is also somewhat counterfactual; in reality, a considerable amount of unsecured debt is extended by creditors. For the benchmark economy studied here, $\pi^a$ primarily determines the level of the debt that individuals can obtain. We thus calibrate $\pi^a$ to replicate the average debt to income ratio observed in the data. Livshits et al. (2007) report that the ratio of unsecured debt to disposable income over the 1995 to 1999 period was 8.4%. Since disposable income over the same period was 72.7% of GDP, we set $\pi^a = 0.14$ to achieve a target debt to income ratio of 6.1%. To put the level of asset garnishment in perspective, it is useful to think of the implicit interest rate that individuals face upon default. In the benchmark economy, the level of $\pi^a$ implies that individuals who defaulted in the past keep around 65% of their interest income.8

---

8This level is substantially higher than the 25% used in Rojas and Urrutia (2007). One reason is that they calibrate their economy under a social security system and perform the opposite experiment, i.e. the removal of the system. As we will see in the next section, debt levels are much lower under a social security system. Hence they need a harsher punishment to sustain any given level of debt, even if their target debt/income ratio is about half of ours—this is simply because the level of unsecured debt has increased substantially from the 80’s—the period they use for their calibration—to the late 90’s.
3.2 Welfare Measures

A government policy regime essentially boils down to the choice of $\tau$. Let $c_j(x, \tau)$ for $j = 1, 2, 3$ be the equilibrium consumption allocation for a type $x$ individual under regime $\tau$ and define the function:

$$W(\lambda, \tau) = \int \sum_{j=1}^{3} \beta^{j-1} u\left((1 + \lambda)c_j(x, \tau)\right) df(x).$$

We interpret $W(0, \tau)$ as the ranking that a ‘representative’ individual (behind the veil of ignorance) would attach to living in an economy under policy regime $\tau$.

Our benchmark economy is that of a laissez-faire regime; i.e., $\tau = 0$. Our ‘compensating variation’ measure $\lambda$ is computed according to:

$$W(\lambda, \tau) = W(0, 0).$$

That is, $\lambda$ has the interpretation of being the fraction of per-period consumption that one would have to compensate the representative individual for moving from the laissez-faire policy regime to the policy regime $\tau$. Accordingly, positive values imply that a representative individual would rather live in the laissez-faire world, and a negative value implies that a representative individual would rather live in a world with the alternative policy regime.

Below, we also present compensating variations conditional on ability. In this case, redefine the function

$$W(x, \lambda, \tau) = \sum_{j=1}^{3} \beta^{j-1} u\left((1 + \lambda)c_j(x, \tau)\right)$$

and compute the type-contingent compensating variation $\lambda(x)$ according to:

$$W(x, \lambda, \tau) = W(x, 0, 0).$$

3.3 Properties of the Calibrated Model

In Table 2, we report a number of statistics produced by our calibrated model across a range of parameter values for $\pi^a$ (where $\pi^a = 0.14$ corresponds to the benchmark
Table 2: Steady States for Different Values of $\pi^a$

<table>
<thead>
<tr>
<th></th>
<th>$\pi^a = 0.00$</th>
<th>$\pi^a = 0.07$</th>
<th>$\pi^a = 0.14$</th>
<th>$\pi^a = 0.20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital/Output</td>
<td>3.086</td>
<td>3.043</td>
<td>3.000</td>
<td>2.964</td>
</tr>
<tr>
<td>Interest Rate (%)</td>
<td>4.452</td>
<td>4.523</td>
<td>4.593</td>
<td>4.652</td>
</tr>
<tr>
<td>Fraction of Debtors</td>
<td>0.000</td>
<td>0.066</td>
<td>0.066</td>
<td>0.066</td>
</tr>
<tr>
<td>Constrained Debtors</td>
<td>1.000</td>
<td>0.924</td>
<td>0.851</td>
<td>0.718</td>
</tr>
<tr>
<td>Debt/Output</td>
<td>0.000</td>
<td>0.031</td>
<td>0.061</td>
<td>0.086</td>
</tr>
<tr>
<td>Compensating Variation (%)</td>
<td>−0.353</td>
<td>−0.194</td>
<td>0.000</td>
<td>0.189</td>
</tr>
</tbody>
</table>

economy). In Table 2, Capital/Output refers to the annualized capital-output ratio; the Interest Rate refers to the annualized real rate of interest; the Fraction of Debtors refers to the fraction of the population that chooses a negative net asset position; Constrained Debtors refers to the fraction of debtors who would like to borrow more (and pay it back), but are debt-constrained in the sense described above; and Debt/Output represents the annualized ratio of outstanding net debt relative to output. Finally, Compensating Variation tells us the fraction by which life-time consumption must be augmented/lowered in order to make the ‘representative’ individual indifferent between living in the steady-state of regime $\pi^a$ relative to the steady state of the benchmark regime, as discussed above.

3.3.1 The Benchmark Model

Column 3 of Table 2 records some of the properties of the benchmark model. By construction, the model delivers a capital/output ratio of 3.0, around the historical average in the United States. The annual real rate of return on capital net of depreciation, at 4.6%, is in the neighborhood of the historical average for the U.S. economy. With a population growth rate of 1% per annum, the steady state capital stock is clearly below its Golden Rule level. The model economy features 6.6% of the adult population with negative net worth; this unsecured debt represents 6.1% of annual GDP, consistent with the evidence in Livshits et al. (2007).

Figure 1 displays the life-cycle properties of consumption, earnings, and income.
across ability quintiles. Relative to the bottom four quintiles, the top quintile experiences very rapid consumption growth between the periods of youth and middle age. This rapid rise relative to the rest of the population is explained by the presence of the binding debt constraints that are more prevalent among this group of high-ability individuals. Not surprisingly, the consumption profiles lack the ‘hump-shaped’ behavior that is commonly reported in the literature (Fernández-Villaverde and Krueger (2007)). However, for the purpose of this paper, what matters is the shape of the income profile relative to the desired consumption profile. As long as income declines more than consumption from middle-aged to old, our qualitative results will hold.

Figure 2 displays consumption, earnings, income and the net asset position across the one hundred different ability types of individuals. The differences in earnings across ability types is very small for the first fifty types, but increases dramatically for the remaining types; this in a direct implication of our log-normal specification for ability, and is supported by the data, as the income distribution has long been known to be approximately log-normal (e.g. Battistin et al. (2007)). As a consequence, for any given age cohort, consumption, earnings, income and net asset positions do not differ by very much at the bottom range of ability types. At the upper end of the ability scale, consumption rises rapidly with ability for the middle aged and for the old. For the young, high-ability types, consumption remains relatively low, which reflects the fact that these are the individuals who are debt-constrained; see also the bottom right panel of Figure 2.

### 3.3.2 Varying the Ability to Seize Assets

The parameter $\pi^a$ governs the ability of creditors to seize the assets of debtors in default and hence influences how tightly debt constraints bind. Table 3 displays the impact of changes in the ability to seize assets under the assumption that prices remain constant at their benchmark levels. Under this assumption, a change in $\pi^a$ only affects individuals’ ability to borrow without changing their desired (unconstrained) asset position. The case of $\pi^a = 0$ corresponds to a situation where debt constraints take the form of an exogenous non-negative net worth restriction. As $\pi^a$ increases, the ability of creditors to seize assets improves and the amount of unsecured debt extended rises. Note, however, that the number of individuals wishing to borrow does not appear to
Figure 1: Life-Cycle Properties of the Benchmark Model Across Ability Types

(a) Quintile I

(b) Quintile II

(c) Quintile III

(d) Quintile IV

(e) Quintile V
be very sensitive to this parameter, at least for the range considered. When general equilibrium effects are ignored, increasing $\pi^a$ allows individuals to better smooth consumption and thus makes them better off.

Table 2 on page 14 reports the general equilibrium results from varying $\pi^a$ between 0.00 and 0.20. Increases in $\pi^a$, which increase levels of net indebtedness, imply lower aggregate saving and stock of capital, and hence a higher interest rate. An important effect of the lower capital stock is the depressing effect it has on real wages. In fact, we see that improvements in the technology for asset seizure actually lead to a steady state with reduced levels of aggregate welfare; this is primarily due to the lower capital
stock (and wages) associated with higher levels of indebtedness. This contrasts with results in Mateos-Planas and Seccia (2006), who find that harsher punishments in a pure exchange economy lead to improved welfare despite the effect on the interest rate, as the effect on the wage rate is not present in their framework.

Figure 3 displays the heterogeneous impact that improvements in the ability to seize assets has across ability types. The top panel of Figure 3 reveals that increases in $\pi^a$ allow the young high-ability types to increase their loan levels substantially; the effect on the desired net asset positions of those with relatively low ability are negligible. While increasing $\pi^a$ results in a steady state in which welfare in reduced for the ‘representative’ individual, the bottom panel of Figure 3 reveals that the steady state welfare effects, when conditioning on type, are quite heterogeneous (the compensating variation is relative to the benchmark economy). When debt constraints bind tightly (say because $\pi^a$ is low), the majority of young persons are better off, because such a situation results in a higher steady state capital stock and higher wages. Not surprisingly, those that are hurt by the tight debt constraints are the high ability types who wish to borrow against their higher future earnings, but are prevented from doing so since they cannot commit to repaying their debt when $\pi^a$ is low.

Table 3: Partial Equilibrium Impact of Changes in $\pi^a$

<table>
<thead>
<tr>
<th></th>
<th>$\pi^a = 0.00$</th>
<th>$\pi^a = 0.07$</th>
<th>$\pi^a = 0.14$</th>
<th>$\pi^a = 0.20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Aggregate Savings</td>
<td>3.086</td>
<td>3.044</td>
<td>3.000</td>
<td>2.962</td>
</tr>
<tr>
<td>Debtors</td>
<td>0.000</td>
<td>0.066</td>
<td>0.066</td>
<td>0.066</td>
</tr>
<tr>
<td>Constrained Debtors</td>
<td>1.000</td>
<td>0.924</td>
<td>0.851</td>
<td>0.717</td>
</tr>
<tr>
<td>Debt/Output</td>
<td>0.000</td>
<td>0.030</td>
<td>0.061</td>
<td>0.088</td>
</tr>
<tr>
<td>Compensating Variation (%)</td>
<td>0.501</td>
<td>0.237</td>
<td>0.000</td>
<td>−0.182</td>
</tr>
</tbody>
</table>
4 The Impact of Social Security and the Payroll Tax

Hubbard and Judd (1987) explore the welfare implications of a social security system financed with a payroll tax in the presence of an exogenous non-negative net worth constraint. Their primary conclusion is that the welfare benefit (cost) of social security falls (increases) significantly when individuals cannot borrow. The goal of this section is to measure the extent to which these welfare calculations are sensitive to the specification of debt constraints. In particular, we wish to examine whether debt constraints that arise endogenously, and may therefore respond to government policy, exacerbate or mitigate the welfare implications of a social security program.

Following Hubbard and Judd (1987), the program under consideration involves a 6% payroll tax. The proceeds of this tax are distributed to members of the old generation in a lump-sum fashion. The analysis proceeds in a number of stages. We begin by investigating partial equilibrium effects; at this stage we fix factor prices at their benchmark levels. We then consider the economic impact of social security when debt constraints are exogenous (XDC case) in the sense that they remain fixed at their benchmark levels; subsequently, debt constraints are allowed to adjust to the new policy regime (EDC case). We then repeat this exercise, allowing for general equilibrium effects. The results are reported in Table 4.
Table 4: Impact of Social Security

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Economy</th>
<th>Partial Equilibrium</th>
<th>General Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>XDC</td>
<td>EDC</td>
<td>XDC</td>
</tr>
<tr>
<td>Output</td>
<td>1.000</td>
<td>0.937</td>
<td>0.938</td>
</tr>
<tr>
<td>Capital/Output</td>
<td>3.000</td>
<td>2.659</td>
<td>2.662</td>
</tr>
<tr>
<td>Interest Rate (%)</td>
<td>4.593</td>
<td>4.593</td>
<td>4.593</td>
</tr>
<tr>
<td>Debtors</td>
<td>0.066</td>
<td>0.066</td>
<td>0.071</td>
</tr>
<tr>
<td>Constrained Debtors</td>
<td>0.851</td>
<td>1.000</td>
<td>0.855</td>
</tr>
<tr>
<td>Debt/Output</td>
<td>0.061</td>
<td>0.065</td>
<td>0.061</td>
</tr>
<tr>
<td>Comp. Variation (%)</td>
<td>0.000</td>
<td>3.927</td>
<td>3.973</td>
</tr>
</tbody>
</table>

4.1 Partial Equilibrium

Under a similar parameterization, but a considerably richer model, Hubbard and Judd (1987) report that the effect of a 6% payroll tax is to reduce the capital/output ratio by 39%. Our model also predicts a sizable, but somewhat more modest decrease of 11%. As far as aggregate quantities go, the results in Table 4 suggest that the size of error one is to make by abstracting from the endogeneity of the debt constraint is likely very small. When the debt constraint is endogenous, the capital/output ratio does not fall by as much, the number of borrowers increases, the number of constrained borrowers rises by less, the level of net indebtedness is slightly smaller, and aggregate welfare falls by more. All of these effects are attributable to the fact that the provision of social security leads to a tightening of debt constraints. The intuition is that the presence of a social security system flattens the income profile of individuals, bringing it closer to their desired path of consumption. As a result, the value of default increases, and so the level of assets which makes individuals indifferent between defaulting and not defaulting increases.

On the other hand, while the average impact on economic well-being is not much affected by the endogeneity of the debt constraint, the top panel of Figure 4 reveals that for the relatively few individuals who are debt constrained, the added welfare

---

9See their Table 2, p. 638.
loss can be sizeable, with compensating variations increasing by over one percentage point for some individuals. Notice that no individual would rather live in a world with social security, as the present value of all individual’s earnings is lower with social security. For low ability individuals, who are not debt constrained, this is the only effect. For high ability individuals, who are constrained, the additional effect can be seen on Figure 4 when the two lines no longer coincide.

### 4.2 General Equilibrium

In a many-period version of the model developed here (but in the absence of debt constraints), Kotlikoff (1979) estimated that the general equilibrium effects of the U.S. social security system led to a decline in the steady-state capital stock anywhere between 10% and 20%. For a 6% payroll tax, Hubbard and Judd (1987) estimate a 46% decline in the capital/output ratio.\(^\text{10}\) With exogenous debt constraints, our model predicts a 17% decline in the capital stock and a 15.7% decline in the capital/output ratio. These results should increase one’s confidence in the ability of our simple model to approximate the quantitative impact of policy reforms on economic aggregates estimated within the context of a richer class of models that often feature

---

\(^{10}\)See their Table 3, p. 640. However, note that for an intertemporal elasticity of substitution parameter equal to 1/2 (\(\sigma = 2\)), their benchmark general equilibrium model features a capital/output ratio of 9.62, which is substantially higher than what is typically measured for the U.S. economy.
many periods, bequests, and uninsured idiosyncratic uncertainty.

As is consistent with the results reported in Hubbard and Judd (1987), the adverse consequences of a payroll-tax financed social security program in the presence of debt constraints is greatly exacerbated when general equilibrium forces come into play. However, the aggregate differences that arise owing to the endogeneity of the debt constraints remain small. With endogenous debt constraints, the steady state level of indebtedness in the economy does shrink considerably (the debt/output ratio is 30% lower in this case), owing to the tightening of credit market conditions. But the impact that this difference makes on the aggregate capital stock is rather small, with endogeneity resulting in a capital stock that is ‘only’ 1.6% larger. The average welfare differences that arise owing to endogeneity of the debt constraint are also quite small, although note that general equilibrium reverses the partial equilibrium results reported above; i.e., with endogenous debt constraints, steady state welfare does not fall by as much under general equilibrium as it does when debt constraints are exogenous (the reverse is true under partial equilibrium).

Once again, however, we note that these aggregate implications mask a considerable amount of differences that occur at the individual level. The bottom panel of Figure 4 displays the compensating variations required across ability types under general equilibrium for both exogenous and endogenous debt constraints. Since we are only comparing utility across steady state, one must interpret these numbers as whether an individual would rather live in an alternative world rather than live in the world described by the benchmark economy. Evidently, the endogeneity of the debt constraint leaves lower ability types better off relative to the case where debt constraints are assumed to be exogenous. These individuals are not directly affected by the increased tightness of the constraint and the resulting (relative) increase in capital implies a higher return for their labor. The high ability individuals also benefit from the increased return to labor, but these gains are swamped by the tighter debt constraints that afflict this set of individuals. At the aggregate level, this differential impact across ability types more or less washes out, leaving us with the conclusion that in terms of aggregate measures of welfare, endogenizing the debt constraint makes little difference.
4.3 Alternative Bankruptcy Code

Given a calibration strategy in which the debt to income ratio is kept constant, setting \( \pi^a \) to zero and choosing \( \pi^w \) to achieve that level of debt results in the exact same steady state as the benchmark economy. Furthermore, the results in this section are essentially identical under this alternative calibration.

5 Conclusions

This paper investigates whether the welfare calculations usually found in the literature with regard to the introduction of a social security system (e.g., Hubbard and Judd (1987)) are sensitive to the specification of the borrowing constraint. We develop an overlapping generations model in which, in the spirit of Kehoe and Levine (1993), debt restrictions arise endogenously due to the existence of inalienable property rights. The legal structure we consider is one in which creditors lay claims to a fraction of a defaulter’s assets (either current or future). Although an improvement in the ability of creditors to seize assets helps debt constrained individuals to obtain credit, it also lowers the level of the capital stock as well as the wage rate, leading to lower aggregate welfare. This is in contrast to pure exchange economies, as in Mateos-Planas and Seccia (2006), where the effect on wages does not arise.

The results of this paper suggest that if one is only interested in the aggregate impact of social security, then an exogenous specification of the borrowing constraint is not a bad approximation. Indeed, results in Rojas and Urrutia (2007) suggest that our results generalize to economies where individuals face uninsurable idiosyncratic uncertainty, not only in steady state but also during the transition. These aggregate results, however, mask important heterogeneous effects at the individual level.
References


25