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ScienceDirect
Economics Letters 99 (2008) 168–171
economics letters
www.elsevier.com/locate/econbase

Price deflators and the estimation of the production function

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Received 1 November 2006; received in revised form 22 May 2007; accepted 22 June 2007
Available online 30 June 2007

Abstract

The use of industry indices to deflate nominal revenues and expenditure in intermediary inputs has been found to lead to lower scale estimates of the production function. This paper proposes a new approach to solve the estimation biases due to the use of industry deflators which relies on the use of the firms' labour cost.

Keywords: Production function; Price deflators

JEL classification: C23; D24

1. Introduction

Most of the available firm-level datasets report nominal revenues and expenditure for materials but they do not report individual data on output and intermediary input prices. This data limitation is a major problem for the estimation of production functions, given that real output and materials cannot be computed but only approximated using industry deflators. Kettle and Griliches (1996) – hereafter KG – are the first to point out that "the practise of using deflated sales as a proxy for real output will, ceteris paribus, tend to create a downward bias in the scale estimate obtained from the production function regressions" (pag. 344). They suggest a solution to determine the true parameters of the production function which relies on the use of the aggregate industry output.

In a companion paper, Ornaghi (2006), I use a unique panel data of manufacturing firms that reports individual prices for output and intermediary inputs to run two parallel estimates. In the first nominal revenues and expenditures in materials are deflated by industry indices (in accordance with standard practice) while in the second I make use of the information on firm-level prices. Results obtained in that paper show that the practice of using industry indices leads to lower scale estimates, thus confirming the original KG critique. But contrary to the KG suggestion, the aggregate industry output does not seem to provide a viable solution to the problem of deflators as it fails to consider that there are asymmetric biases in the estimated coefficients of inputs. Indeed, that paper shows that the use of industry deflators for expenditure in materials exacerbates the bias in the estimated labour coefficients (already) introduced by the use of industry deflators for revenues, but it offsets the bias in the estimated coefficients for materials.[1]

This paper defines an alternative approach to the estimation of a production function that can be used when firm-level prices are not available. This approach allows obtaining unbiased estimates of the coefficients of labour and materials and results consistent with constant return to scale. Two important caveats to this method deserve mention. First, empirical results show that the problem of small and statistically insignificant coefficient of capital is invariant to the use of firm-level or aggregate deflators. The present analysis does not provide any important

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[1] The importance of using an appropriate deflator for expenditure in intermediary inputs (and not only for revenues) is confirmed by the results in Mairesse and Jaumandreu (2005). Using the same data, they define two specifications which include the same measure of materials and defer only for the measured output, and they find only minor differences between the two estimations.

0165-1765/$ - see front matter
doi:10.1016/j.econlet.2007.06.020

advance towards the understanding of this puzzle. Second, the method proposed in this paper is applicable to specifications in differences where any unobservable component that is persistent over time is purged from the residual. The application to specifications in levels might be problematic, because of identification problems.[2]

2. Econometric framework

As in KG, let me assume for simplicity that the production function of firm i in year t is a Cobb–Douglas function that can be written in a log-linear form as:

$$q_{it} = \alpha_1 l_{it} + \alpha_2 m_{it} + \alpha_3 k_{it} + u_{it} \tag{1}$$

where q, l, m and k are, respectively, the logs of the quantity produced, labour, materials and capital; u is the random error term for the equation. The error term u is assumed to include unobservable firm-specific factors of production, μ_i (e.g. entrepreneurial ability) that determine persistent productivity differences between firms over time. Consequently, the error term can be decomposed as $u_{it}=\mu_i+v_{it}$, where v is a white noise disturbance term.

Taking logarithms and first differences to eliminate μ_i from the residual, we obtain the linear equation:

$$\tilde{q}_{it} = \alpha_1 \tilde{l}_{it} + \alpha_2 \tilde{m}_{it} + \alpha_3 \tilde{k}_{it} + \tilde{v}_{it} \tag{2}$$

where lowercase letters with a tilde stand for logarithmic differences between year $t-1$ and t (e.g. $\tilde{q}_{it}=\ln Q_{it}-\ln Q_{it}-1$).

When firm-level prices for output P_{it} and intermediary inputs G_{it} are not available and the econometrician decides to use the industry indices P_{It} and G_{It}, Eq. (2) is replaced by a new specification:

$$\tilde{y}_{it} = \alpha_1 \tilde{l}_{it} + \alpha_2 \tilde{n}_{it} + \alpha_3 \tilde{k}_{it} + \underbrace{(\tilde{p}_{it} - \tilde{p}_{It}) - \alpha_2(\tilde{g}_{it} - \tilde{g}_{It}) + \tilde{v}_{it}}_{\varepsilon_{it}} \tag{3}$$

where $\tilde{y}$ and $\tilde{n}$ refers to the (log differences of) revenues and expenditure in intermediary inputs deflated by the industry indices, i.e. $\tilde{y} = \ln\frac{P_{it}*Q_{it}}{P_{It}} - \ln\frac{P_{it-1}*Q_{it-1}}{P_{It-1}}$ and $\tilde{n} = \ln\frac{G_{it}*M_{it}}{G_{It}} - \ln\frac{G_{it-1}*M_{it-1}}{G_{It-1}}$; following this transformation, the new disturbance term ε embodies the deviations of individual output and intermediary price from the corresponding industry deflators, $(\tilde{p}_{it}-\tilde{p}_{It})-\alpha_2(\tilde{g}_{it}-\tilde{g}_{It})$ (see Ornaghi, 2006, for details). As the price of output and intermediary inputs affect the optimal choice of production factors, the regressors of Eq. (3) are correlated with ε and this causes biases in estimating their coefficients. Hereafter, I suggest a solution to this problem which consists in replacing the (usually) unknown expression $(\tilde{p}_{it}-\tilde{p}_{It})-\alpha_2(\tilde{g}_{it}-\tilde{g}_{It})$ with a measure of labour costs.

Assume that firms use the price setting rule $P_{it}=\Lambda_i * \mathrm{MC}_{it}$, where MC is the marginal cost and Λ is the constant mark-up applied by the firm. Taking the log first differences, we have:

$$\tilde{p}_{it} = (\ln \mathrm{MC}_{it} - \ln \mathrm{MC}_{it-1}). \tag{4}$$

Solving the cost minimization problem of firm i:

$$\min W{*}L + G{*}M + R{*}K \quad \text{s.t.} \quad Q = L^{\alpha_1} M^{\alpha_2} K^{\alpha_3}$$

where W, G and R indicate the price of labour, materials and the rental price of capital, respectively, we obtain the firm cost function:

$$C(W,G,R,Q) = \gamma{*}Q^{\frac{1}{\alpha_1+\alpha_2+\alpha_3}} W^{\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}} G^{\frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}} R^{\frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}} \tag{5}$$

where γ is a constant that depends on the αs' coefficients of the Cobb–Douglas production function. If returns to scale are constant (hypothesis supported by our estimates), the marginal cost can be written as $\mathrm{MC}=c * W^{\alpha_1}G^{\alpha_2}R^{\alpha_3}$. Taking the first log differences of the latter equation, i.e. $\ln \mathrm{MC}_{it}-\ln MC_{it-1}=\alpha_1\tilde{w}_{it}+\alpha_2\tilde{g}_{it}+\alpha_3\tilde{r}_{it}$, and substituting it into Eq. (4), we obtain that:

$$\tilde{p}_{it} - \alpha_2 \tilde{g}_{it} = \alpha_1 \tilde{w}_{it} + \alpha_3 \tilde{r}_{it}\,. \tag{6}$$

The unknown prices of output and intermediary inputs are now expressed in terms of the price of labour (generally observable) and the rental price of capital. Following the approach used in Klette (1999), this latter equation can be expressed in terms of deviation from a reference point, which can be thought of as the price of output and inputs of the representative firm. Empirically, this reference point is computed as the average value of the relevant variables within the industry. After normalizing with respect to these industry indices, Eq. (6) can be restated as:

$$(\tilde{p}_{it} - \tilde{p}_{It}) - \alpha_2(\tilde{g}_{it} - \tilde{g}_{It}) = \alpha_1(\tilde{w}_{it} - \tilde{w}_{It}) + \alpha_3(\tilde{r}_{it} - \tilde{r}_{It}). \tag{7}$$

Finally, by replacing Eq. (7) into Eq. (3), we get:

$$\tilde{y}_{it} = \alpha_1 \tilde{l}_{it} + \alpha_2 \tilde{n}_{it} + \alpha_3 \tilde{k}_{it} + \alpha_1(\tilde{w}_{it} - \tilde{w}_{It}) + \underbrace{\alpha_3(\tilde{r}_{it} - \tilde{r}_{It}) + \tilde{v}_{it}}_{\varepsilon_{it}} \tag{8}$$

The panel-data used in this study reports all the relevant information for computing the cost of labour.[3] As rental price of capital depends on the individual rate of return required by a firm, the price of investment goods and the tax treatment (see Hall and Jorgenson, 1969, among others), most of which are not observable, I opt for not computing a proxy for the cost of capital. The term $(\tilde{r}_{it}-\tilde{r}_{It})$ enters then the error term ε. It must be considered that, in the absence of reliable data, the residual would embody the measurement errors of the rental cost of capital anyway.

[2] This is unfortunate since the information contained in levels have been used to attenuate the bias in the estimation of the capital coefficient, as in the paper by Blundell and Bond (2000).

[3] The value of W for each firm is computed divided the total cost of labour by the number of employees.

Table 1
Production function estimates

	I	II	III
	Eq. (2)[a]	Eq. (3)[a]	Eq. (8)[a]
Labour	0.324***	0.187***	0.310***
	(0.068)	(0.065)	(0.073)
Materials	0.616***	0.669***	0.615***
	(0.047)	(0.050)	(0.052)
Capital	−0.043	−0.118**	−0.063
(c.r. to s.)	(0.063)	(0.061)	(0.058)
Wage	–	–	0.344**
			(0.150)
Ind Dum	Inc.	Inc.	Inc.
Time Dum	Inc.	Inc.	Inc.
Period	1990–99	1990–99	1990–99
N. obs	11,476	11,476	11,476
Sargan T (*df*)	55.4 (55)	54.6 (55)	57.2 (55)
m1	−9.54	−9.56	−9.47
m2	−0.25	−0.60	−0.99

Estimation method: GMM estimates. Heteroskedasticity robust standard errors shown in parentheses. *Significant at 10% level; **significant at 5%; ***significant at 1%.

[a] *IV's*: number of workers, physical capital and materials lagged levels from $t-2$ to $t-4$; (exogenous variables) growth of capital.

There are two reasons why the bias in estimating the parameters of specification Eq. (8) can be expected to be smaller compared to specification Eq. (3). First, the conditional factor demands for labour and materials derived from the cost minimization problem above respond more to the price of output and intermediary inputs than to the rental cost of capital. Second, to the extend that the rental cost of capital is rather constant over time or it has a negligible dispersion within the firms in the industry, the noise term $(\tilde{r}_{it}-\tilde{r}_{It})$ is largely captured by the firm specific effect and industry dummies. Results presented in the following section seem to confirm this perspective.

Before discussing the empirical results, a final remark is required. If the assumption of constant markups does not hold, the error term in Eq. (8) would also include an unobserved term in differences of the markups. Under this alternative scenario, estimated coefficients might be biased because of a possible correlation of this unobservable with factor quantities. By comparing the estimates of Eqs. (2) and (8), I find that there are no relevant bias due to the *omitted markups variable*. The question now is: How can one be sure that estimates are not biased when a similar comparison cannot be done? The econometric framework defined above suggests that a formal test of the null hypothesis that the coefficients of labour and wages in Eq. (8) are equal may help to address this question. Given that the correlation of labour and wages with any unobserved variable (e.g. markups or cost of capital) can go in different directions or at least, have different intensities, failing to reject the null hypothesis would provide some support in favour of the approach proposed in this paper.[4]

[4] It is important to note that the existing literature about the cyclical behaviour of markups and real wages is filled with conflicting hypothesis and inconclusive empirical evidence. This seems to confirm that there are no theoretical reasons or empirical evidence to suspect a bias in the proposed approach.

3. Regression results

Variables are computed using data retrieved from the *Encuesta sobre Estrategias Empresariales*, ESEE, (Business Strategy Survey): an unbalanced panel sample of Spanish manufacturing firms covering the period 1990–1999. Details about the panel-data sample and the variables used for the estimation can be found in Ornaghi (2006).

The specifications estimated are those reported in Eqs. (2), (3) and (8) above. For interpretive reasons, all these equations are restated so that the hypothesis of constant returns can be tested explicitly (i.e. H_0: $\gamma=1$ where $\gamma\equiv\alpha_1+\alpha_2+\alpha_3$). For instance, Eq. (2) is actually specified as:

$$\tilde{q}_{it}-\tilde{k}_{it}=\alpha_1(\tilde{l}_{it}-\tilde{k}_{it})+\alpha_2(\tilde{m}_{it}-\tilde{k}_{it})+(\gamma-1)\,\tilde{k}_{it}+\tilde{v}_{it}\,.$$

The estimation method is the generalized method of moment (GMM) applied to panel data. The set of instrumental variables are held fixed across the specification presented below, so that any differences in estimates can be attributed to the change in the variables used.

Table 1 presents the estimated coefficients of the three different specifications. Column I refers to specification Eq. (2) in which I use real measures of output and intermediary inputs; column II corresponds to Eq. (3) based on industry deflators while specification in column III refers to Eq. (8), where the term $(\tilde{w}_{it}-\tilde{w}_{It})$ is added to soften the problem created by industry deflators. Results in the first two columns are equivalent to those reported in Table 1 of Ornaghi (2006). They show that scale elasticities are lower when deflated sales, *Y*, and expenditure in intermediate inputs, *N*, are used instead of output, *Q*, and materials, *M*. In particular, the null hypothesis of constant return to scale is rejected at 5% significant level only in column II. The new interesting finding of this paper is that estimates are notably improved when we use information on labour costs (column III): the hypothesis of constant return to scale cannot be rejected and the estimated elasticity of labour is similar to the value obtained in column I. Moreover, the coefficient of wage is very close to that of labour; the null hypothesis that the two coefficients are not statistically different cannot be rejected at any level of significance (the *p*-value of the Wald-test statistic is 0.72), thus giving further support to the proposed approach.[5]

To check whether the differences between the estimated coefficients of the variables reported in Table 1 are statistically significant, I compute bootstrap confidence interval by running 400 replications of the *GMM* estimator.[6] Table 2 shows the

[5] As our empirical specifications are defined to test the hypothesis of c.r.s., the well-known problem of small capital coefficient is not immediately apparent. However, it is easy to see that the capital coefficient takes low values across all the specifications. As already said, our estimates leave this puzzle unsolved. Nevertheless, the results of this paper are not undermined by this problem. By defining an alternative measure of capital using data on capacity utilization, it is possible to get point estimates of the capital coefficient that are five times higher than those reported in Table 1 (see Ornaghi, 2003, for further details). The use of this "improved" measure of capital does not change the finding of this paper with respect to the estimation bias of industry deflators.

[6] Note that this estimation are based on the "block" bootstrapping procedure. This consists in randomly drawing a sample of firms and, for each drawn firm, all the yearly observations available are used, i.e. the observations of a given firm are kept together.

Table 2
Test of differences in coefficients

Variables	IV	V
	Eq. (2) vs. Eq. (3)	Eq. (2) vs. Eq. (8)
Labour	0.117	0.011
	[0.065; 0.166]	[−0.094; 0.098]
Materials	−0.044	−0.001
	[−0.082; −0.011]	[−0.051; −0.054]
Capital	0.065	0.015
(c.r. to s.)	[0.020; 0.106]	[−0.055; 0.072]

Values reported are the bootstrap average differences and, in brackets, the 90% confidence interval based on 400 replications of the GMM estimations.

average differences of the estimated coefficients together with 90% confidence interval in square brackets. Column IV reports the difference between Eqs. (2) and (3) (as in Table 2 of Ornaghi, 2006) while column V those between Eqs. (2) and (8). The confidence intervals confirm that there are not significant differences between estimation based on firm-level deflators and on industry deflators when the wage term is included in the specification.

Acknowledgement

The author would like to thank an anonymous referee for useful comments.

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