

# ASSESSING THE EFFECTS OF MEASUREMENT ERRORS ON THE ESTIMATION OF PRODUCTION FUNCTIONS

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## SUMMARY

This article explores to what extent the poor results that are often found when estimating parameters of production functions can be attributed to measurement errors, due to the use of common price deflators across firms. Because of the lack of detailed micro-economic data, econometricians have to rely on industry-wide deflators when computing outputs and intermediate inputs. A unique feature of the longitudinal data used in this paper is that it reports firm-level prices. This allows for a comparative assessment of production function parameters where the outputs and intermediate inputs are computed using both firm-specific prices and industry-wide deflators. The empirical results presented in this paper show that the use of common deflators across firms leads to lower scale estimates, mainly because of a large downward bias in the estimated coefficients for labour. Copyright © 2006 John Wiley & Sons, Ltd.

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## 1. INTRODUCTION

Despite the existence of a wide literature dealing with empirical estimation of production functions, there are some interesting and challenging questions that have not been solved. Although simple OLS regression gives plausible parameter estimates that are in line with factor shares and consistent with constant return to scale, econometricians agree that the usual exogeneity assumptions of the regressors that are required for the consistency of OLS are unlikely to hold. To the extent that a productivity shock is anticipated before the optimal quantity of inputs is chosen, the production function disturbances are transmitted to the inputs demand equation. This can lead to an estimation bias due to the correlation between the right-hand variables and the error term (often defined as ‘simultaneity bias’). Moreover, it is likely that each firm is characterised by specific factors of production, such as entrepreneurial ability, that are not observable but that can affect the current inputs choice, thus worsening the bias in OLS estimates.

The availability of firms’ panel data provides the necessary tools to get around the strict exogeneity requirements of the regressors. Under the assumption that productivity differences among firms tend to be rather persistent over time, firm-specific effects can be eliminated by specifying the model in differences instead of levels. At the same time, panel data provides the necessary set of instruments to correct for simultaneity in these first differenced equations. The most widely used technique is the Generalized Method of Moments (GMM), which relies on combining in an optimal way a set of orthogonal conditions generally defined using suitably lagged levels of the inputs as instruments.<sup>1</sup>

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<sup>1</sup> See Arellano and Bond (1991) for a detailed discussion.

Quite surprisingly, less satisfactory results are usually obtained when controlling for unobserved heterogeneity and simultaneity: low and often insignificant capital coefficients and unconvincing low estimates of returns to scale. Several explanations for the problems endemic to the estimators in differences have been proffered. Blundell and Bond (2000) suggest that these problems stem from the weak correlation that exists between the growth rates of capital and employment and the lagged levels of these inputs. Being that the capital and the employment series are highly persistent, past levels of these variables contain little information about the specification in first differences. They show that estimation can be notably improved by adding lagged first differences as instruments for the equation in levels. Using this ‘extended’ GMM estimator, they find a higher and significant coefficient for capital and fail to reject the constant returns to scale hypothesis.<sup>2</sup> The explanation and proposed solution both reside within the econometric field.<sup>3</sup>

On the contrary, Klette and Griliches (1996)—hereafter KG—suggest that the implausible low estimates of returns to scale can be explained by the mis-measurement of output. They affirm that ‘the practice of using deflated sales as a proxy for real output will, *ceteris paribus*, tend to create a downward bias in the scale estimate obtained from the production function regressions’ (p. 344). They stress that the estimated parameters in production function analysis are reduced form parameters, which are derived from the interaction of the production function with the demand equation. True elasticities of scale can only be gained through the modelling of a more general framework where the production function is augmented with a demand equation and a price-setting rule. The authors also present an empirical estimation to illustrate their theoretical approach. Unfortunately, without the availability of firm-level data, it is impossible to know the magnitude of the bias in scale estimates due to the use of common price deflators, and to determine whether reinterpreting the parameters as reduced form coefficients actually provides a reliable solution to this bias.

The aim of this paper is to provide some evidence on the magnitude of the bias due to the use of common price deflators across firms. The database used reports variations in output prices at the firm level. This allows for the construction of two measures of output: first, deflating revenues using firm-level price data and, second, using industry-wide deflators (the measure generally used as proxy for output).<sup>4</sup> As suggested by KG, the results reported in Section 4 show that scale estimates are downward biased when using industry-wide deflated revenues (or value-added) to compute the dependent variable of a production function. Additionally, evidence is provided on another aspect that has gone largely unnoticed in this literature: the fact that the difficulty pointed out for deflated revenues needs to be extended to the computation of materials. Given that most of the available data do not report a ‘quantity’ measure of materials, the general approach is to deflate the normal expenditure in intermediate inputs by a common deflator across firms. The underlying hypothesis made is that there is perfect competition in this factor market, so that all the firms are charged with the same price. However, significant dispersion in input prices

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<sup>2</sup> The ‘extended’ GMM estimator was first proposed by Arellano and Bover (1995). Blundell and Bond (2000) apply this ‘extended’ GMM estimator to the estimation of production functions. Additional references which consider this ‘extended’ GMM estimator to the estimation of production functions are Blundell *et al.* (2000) and Alonso-Borrego and Sanchez-Mangas (2001).

<sup>3</sup> Note that another simple explanation of why unreasonable results are obtained using fixed-effects estimators is that the unobserved firm effect is not constant over time, so that it cannot be removed by taking first differences.

<sup>4</sup> In a recent paper, Eslava *et al.* (2004) use firm-level data on prices for outputs and inputs to overcome the bias in the estimation of total factor productivity when using industry-wide deflators.

seems to exist between firms in the same industry.<sup>5</sup> The theoretical part of the paper shows that the use of deflated expenditure in materials may exacerbate the bias in the estimated labour coefficients (already) introduced by the use of deflated revenues, but it may offset the bias in the estimated coefficients for materials. The empirical results confirm the existence of an asymmetric bias among inputs coefficients. The immediate implication of this result is that it invalidates the solution advanced by KG as this relies on applying an identical 'upward' correction to all inputs coefficients.

The empirical part of the paper is based on a micro panel dataset of Spanish manufacturing firms that includes about 2000 entities during the period 1990–1999. In addition to the standard data on firms' revenues and factors of production, this unique dataset reports the prices for outputs and intermediate inputs that permit the computation of a precise measure of 'physical' output and materials.

This paper does not consider the problem of changes in the quality attributes of output and inputs. This choice can be justified both on empirical and theoretical grounds. As a matter of fact, accounting for quality changes requires the use of hedonic prices, which are generally not available and rather difficult to construct. I prefer then to rely on the (available) firm-level prices to compute a 'physical' measure of the output. On a theoretical ground, there is not a general agreement of whether or not the output  $Q$  has to be adjusted for quality changes. Greenwood *et al.* (1997) affirm that 'there is a controversy in the growth-accounting literature over whether or not GNP should be adjusted upwards to reflect quality improvements in new capital goods. The general equilibrium approach taken here provides a decisive answer to this question: it should not be'. The empirical results presented below suggest that interesting findings on the research questions here addressed can be obtained, even without taking into account the issue of quality changes.<sup>6</sup>

The structure of this paper is as follows: Section 2 provides a general theoretical analysis of the bias in the production function regression when using deflated revenues and deflated expenditure in intermediate inputs. Section 3 describes the data and explains how the variables are constructed. Section 4 discusses the empirical results, while Section 5 draws some final comments.

## 2. THEORETICAL AND ECONOMETRIC FRAMEWORK

This section provides a set of theoretical underpinnings to clarify the consequences of using incorrect measures of output and factors of production. KG show that if the unobserved prices are correlated with the explanatory variables included in the model, an omitted variable bias will arise. The same analysis needs to be extended to intermediate input prices: using common deflators for expenditure in materials can exacerbate or reduce the bias mentioned above.

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<sup>5</sup> Although the assumption of sticky intermediate inputs prices was supported by some empirical evidence in the 1980s and 1990s, recent empirical work has departed from this view. In a recent article based on unpublished data of the Bureau of Labour Statistics, Bils and Klenow (2004) observe 'much more frequent price changes than reported in most previous studies, with half of goods displaying prices that last 4.3 months or less'. At the same time, looking at data on changes in input prices in a large panel data of US manufacturing firms, Beaulieu and Matthey (1999) find large variation in idiosyncratic cost shocks.

<sup>6</sup> Note that this work is inspired by the article of Klette and Griliches (1996) where quality is not explicitly considered.

## 2.1. Deflators Bias

To keep things as simple as possible, assume that the production function for manufacturing firms can be represented by a Cobb–Douglas function in three inputs: labour  $L$ , physical capital  $K$  and materials  $M$ .<sup>7</sup>

$$Q_{it} = L_{it}^{\alpha_1} M_{it}^{\alpha_2} K_{it}^{\alpha_3} e^{\vartheta_{it}^p} \quad (1)$$

where  $Q$  is the quantity produced and  $\vartheta^p$  is the random error term for the equation.<sup>8</sup> One of the most important components of  $\vartheta^p$  is likely to be due to firm-specific factors of production, such as entrepreneurial ability, that are not observable. This component determines productivity differences between firms that tend to be rather persistent over time. Consequently, the error term can be decomposed as  $\vartheta_{it}^p = \mu_i + u_{it}^p$ , where  $\mu_i$  is the time-invariant term just mentioned that accounts for the permanent heterogeneity across firms, whereas  $u^p$  includes temporary productivity shocks and measurement errors.<sup>9</sup>

Taking logarithms and first differences to eliminate  $\mu_i$  from the residual, we obtain the following linear equation:

$$\tilde{q}_{it} = \alpha_1 \tilde{l}_{it} + \alpha_2 \tilde{m}_{it} + \alpha_3 \tilde{k}_{it} + \tilde{u}_{it}^p \quad (2)$$

where lower-case letters with a tilde stand for logarithmic differences between year  $t - 1$  and  $t$  (e.g.,  $\tilde{q}_{it} = \ln Q_{it} - \ln Q_{it-1}$ ). Assume that available data includes revenues and an industry-wide deflator, which is used to construct deflated revenues,  $Y$ , as a proxy for real output,  $Q$ . Then, the relationship between output growth,  $\tilde{q}$ , and deflated revenues growth,  $\tilde{y}$ , is given by

$$\tilde{q}_{it} + \tilde{p}_{it} - \tilde{p}_{It} = \tilde{y}_{it} \quad (3)$$

where  $\tilde{p}_{it}$  is the growth in firm  $i$ 's specific price, while  $\tilde{p}_{It}$  is the growth in the industry-wide deflator.<sup>10</sup>

Substituting (3) in equation (2), we get

$$\tilde{y}_{it} = \alpha_1 \tilde{l}_{it} + \alpha_2 \tilde{m}_{it} + \alpha_3 \tilde{k}_{it} + (\tilde{p}_{it} - \tilde{p}_{It}) + \tilde{u}_{it}^p \quad (4)$$

The latter equation shows that when deflated revenues replace the 'physical' measure of output the omitted price variable  $(\tilde{p}_{it} - \tilde{p}_{It})$  enters into the residual term. As long as the optimal choice of factors of production is affected by changes in the firm's prices, the omitted price term tends to create a bias in OLS estimator of the parameter vectors  $\alpha$ .<sup>11</sup> At the same time, if the variable observed does not represent a 'quantity' measure of materials,  $M$ , but rather expenditure in materials deflated with an industry-wide deflator,  $N$ , then we have that  $\tilde{n}_{it} = \tilde{m}_{it} + \tilde{g}_{it} - \tilde{g}_{It}$ ,

<sup>7</sup> As stressed by Griliches and Mairesse (1984, p. 342), 'one could, of course, consider more complicated functional forms, such as the CES or Translog functions. . . this will not matter as far as our main purpose of estimating the output elasticities of R&D and physical capital, or at least their relative importance, is concerned'.

<sup>8</sup> Subscripts are reported only when it is strictly necessary in order to simplify notation.

<sup>9</sup> The term  $u^p$  is assumed to be an uncorrelated zero mean error term. This implies that disturbances of specifications in first differences are expected to present negative first-order autocorrelation and absence of autocorrelation of higher orders. These assumptions are supported by the  $m1$  and  $m2$  statistics for serial correlation reported in the tables of Section 4.

<sup>10</sup> Suppose that we have data on revenues,  $P_{it} * Q_{it}$ , and we use a common price deflator  $P_{It}$  to get deflated revenues  $Y_{it} = (P_{it} * Q_{it})/P_{It}$ . Taking logarithms and first differences, we obtain  $\tilde{y}_{it} = \tilde{q}_{it} + \tilde{p}_{it} - \tilde{p}_{It}$ .

<sup>11</sup> Let us define the term  $(\tilde{p}_{it} - \tilde{p}_{It})$  as  $\Pi_{it}$ . As shown in Klette and Griliches (1996, p. 346), the bias in the OLS estimator of the  $\alpha$  coefficients depends on the sign and the magnitude of the parameter  $\delta$  in the auxiliary regression:  $\Pi = X\delta + u^\Pi$ , where  $X$  is the matrix of input growth rates and  $u^\Pi$  is a white-noise error term.

where  $\tilde{g}_{it}$  and  $\tilde{g}_{It}$  are (log differences of) intermediate input prices at firm level and aggregate level, respectively.<sup>12</sup> Accordingly, equation (4) should be rewritten as follows:

$$\tilde{y}_{it} = \alpha_1 \tilde{l}_{it} + \alpha_2 \tilde{n}_{it} + \alpha_3 \tilde{k}_{it} + (\tilde{p}_{it} - \tilde{p}_{It}) - \alpha_2 (\tilde{g}_{it} - \tilde{g}_{It}) + \tilde{u}_{it}^p \quad (5)$$

The term  $(\tilde{g}_{it} - \tilde{g}_{It})$  can either exacerbate or offset the bias introduced by the output price variable  $(\tilde{p}_{it} - \tilde{p}_{It})$ .<sup>13</sup> The difficulties outlined above may not be solved by using appropriate instrumental variables. Potentially useful instruments which have to be correlated with the input growth rates (e.g., lagged inputs) are likely to be correlated with inputs and output price changes buried in the residual, and if so are illegitimate instruments. Our results, obtained using the instrumental variable method, seem to confirm this conjecture.

Following the analysis presented in KG, the consequences of using common deflators on estimating the production function parameters can be illustrated defining a log-linear demand equation for firm  $i$  of the following type:

$$Q_{it} = Q_{It} * (P_{it}/P_{It})^\eta e^{u_{it}^d} \quad (6)$$

where  $Q_{it}$  stands for firm  $i$ 's sales,  $Q_{It}$  is the industry output,  $P_{it}$  is the firm-specific price, while  $P_{It}$  is the average price in the industry. Equation (6) shows that the firm output is jointly determined by the price of the product relative to the industry index and the aggregate industry sales.<sup>14</sup> Rewriting the demand system in terms of growth rates (more precisely as first differences of logarithms), we obtain

$$\tilde{q}_{it} = \tilde{q}_{It} + \eta(\tilde{p}_{it} - \tilde{p}_{It}) + \tilde{u}_{it}^d \quad (7)$$

Using equation (3) and rearranging yields

$$(\tilde{p}_{it} - \tilde{p}_{It}) = \frac{1}{1 + \eta} (\tilde{y}_{it} - \tilde{q}_{It} - \tilde{u}_{it}^d) \quad (8)$$

Equation (8) gives an analytical interpretation of the (usually) unobserved term  $(\tilde{p}_{it} - \tilde{p}_{It})$ . Apart from the residual, the only variable that is unknown to the econometrician is the industry output growth  $\tilde{q}_{It}$ . KG show that it is possible to use the weighted average growth in deflated revenues of all the firms in the sample as a proxy for this term. Combining equations (5) and (8), we get

$$\tilde{y}_{it} = \frac{\eta + 1}{\eta} (\alpha_1 \tilde{l}_{it} + \alpha_2 \tilde{n}_{it} + \alpha_3 \tilde{k}_{it}) - \frac{1}{\eta} \tilde{q}_{It} - \alpha_2 \frac{\eta + 1}{\eta} (\tilde{g}_{it} - \tilde{g}_{It}) + u_{it} \quad (9)$$

where  $u_{it}$  is an error term that captures both demand and productivity shocks.

Equation (9) highlights the two main points remarked by KG. First, the estimated parameters for the production function have to be interpreted as reduced form parameters (stemming from supply and demand coefficients). Second, the growth rate in industry output  $\tilde{q}_{It}$  can assure identification

<sup>12</sup> The price deflators for expenditures in intermediate inputs used in this study is a Paasche-type price index computed from the percentage variations in the prices of purchased materials, energy and services reported by the firms. This can be considered a precise measure of the price paid by firms for different intermediate inputs.

<sup>13</sup> Define the term  $(\tilde{g}_{it} - \tilde{g}_{It})$  as  $\Omega_{it}$ . Using a symmetric argument to the one in footnote 11, we have that the OLS estimator of the parameter vector  $\alpha$  depends also on the auxiliary regression:  $\Omega = X\gamma + u^\Omega$ . Then, the overall direction and size of the bias depend on the sign and magnitude of  $\delta$  and  $\gamma$ .

<sup>14</sup> A similar demand system has been widely examined in the industrial organization literature under the label 'the Spence-Dixit-Stiglitz' model. See Klette and Griliches (1996) and Klette (1996) for further details.

of standard inputs coefficients since the demand elasticity  $\eta$  can be inferred from the estimated coefficient of this variable.

However, equation (9) goes further than the analysis of KG as it includes the omitted materials price,  $(\tilde{g}_{it} - \tilde{g}_{It})$ .<sup>15</sup> It is likely that firms that experience higher costs of materials will, *ceteris paribus*, reduce the quantity used of this input in favour of the other short-run factors of production, such as labour. In econometric terms, this is equivalent to assuming the existence of a systematic negative correlation of the generally unobserved term  $(\tilde{g}_{it} - \tilde{g}_{It})$  with materials and a positive correlation with labour. Given the negative sign of the  $(\tilde{g}_{it} - \tilde{g}_{It})$  coefficient, the downward bias in the estimate of input elasticities due to the omitted output price  $(\tilde{p}_{it} - \tilde{p}_{It})$  is exacerbated in the case of labour but it is (partially or fully) offset for materials.<sup>16</sup> The asymmetric bias that the term  $(\tilde{g}_{it} - \tilde{g}_{It})$  may introduce in the estimates of input coefficients can invalidate the solution suggested by KG since this applies an identical upward correction,  $\eta/(\eta + 1)$ , to all factors of production. The empirical results shown in Section 4 seem to confirm this perspective.

## 2.2. Input Elasticities and Scale Economies

This section presents the empirical specifications to be estimated. All the models are defined using, first, a ‘quantity’ measure of output,  $Q$ , and materials,  $M$  (that is, the generally not observable measures) and, second, deflated revenues,  $Y$ , and deflated expenditure in intermediate inputs,  $N$  (the variables generally used in most empirical analysis). All of these specifications provide interesting evidence on the extent of the bias due to the mis-measurement of the variables.

Consider equation (2) above. Under constant returns to scale with respect to standard factors of production (labour, materials and physical capital),  $\alpha_1 + \alpha_2 + \alpha_3 = \varepsilon = 1$  holds. For interpretive reasons, equation (2) is restated so that deviations from constant returns are measured explicitly:

$$\tilde{q}_{it} - \tilde{k}_{it} = \alpha_1(\tilde{l}_{it} - \tilde{k}_{it}) + \alpha_2(\tilde{m}_{it} - \tilde{k}_{it}) + (\varepsilon^{\text{ob}} - 1)\tilde{k}_{it} + \tilde{u}_{it}^{\text{ob}} \quad (\text{S1})$$

The corresponding specification using deflated variables is

$$\tilde{y}_{it} - \tilde{k}_{it} = \gamma_1(\tilde{l}_{it} - \tilde{k}_{it}) + \gamma_2(\tilde{n}_{it} - \tilde{k}_{it}) + (\varepsilon^{\text{unob}} - 1)\tilde{k}_{it} + \tilde{u}_{it}^{\text{unob}} \quad (\text{S2})$$

<sup>15</sup> This point is recognized in the article of KG when they affirm that ‘measurement errors in the input index will cause a bias in the OLS regression’ (p. 355) but they do not address the problem explicitly.

<sup>16</sup> Suppose that we have a ‘physical’ measure of output,  $Q$ . Moreover, we know total expenditure in intermediate inputs but we do not have individual prices charged to the firm for these inputs; this means that we can construct a proxy for materials using general deflators. Then, the (first-differenced) specification of our production function would be:  $\tilde{q}_{it} = \alpha_1\tilde{l}_{it} + \alpha_2\tilde{n}_{it} + \alpha_3\tilde{c}_{it} - \alpha_2(\tilde{g}_{it} - \tilde{g}_{It}) + \tilde{u}_{it}$ . Let us define  $(\tilde{g}_{it} - \tilde{g}_{It})$  as  $\Omega_{it}$  and assume that the error term is uncorrelated with the inputs (i.e., there is no bias from transmission of productivity shocks). The probability limit of the OLS estimator of the labour and material coefficients would be

$$\begin{aligned} \text{plim}(\hat{\alpha}_1) &= \frac{\text{cov}(\tilde{l}_{it}, \tilde{q}_{it})}{\text{var}(\tilde{l}_{it})} = \alpha_1 - \alpha_2 \frac{\text{cov}(\Omega, \tilde{l}_{it})}{\text{var}(\tilde{l}_{it})} \\ \text{plim}(\hat{\alpha}_2) &= \frac{\text{cov}(\tilde{n}_{it}, \tilde{q}_{it})}{\text{var}(\tilde{n}_{it})} = \alpha_2 - \alpha_2 \frac{\text{cov}(\Omega, \tilde{n}_{it})}{\text{var}(\tilde{n}_{it})} \end{aligned}$$

It is likely that firms that experience a higher growth in prices of intermediate inputs,  $\Omega_{it}$ , will try to substitute them with labour. For example, firms can decide to reduce the ‘outsourcing’ of their activities when the cost of performing these activities inside is less than contracting them out. This reasoning implies a positive value of the covariance term in the first equation reported above and a negative value for the corresponding term in the second equation. Then, we can expect that  $\hat{\alpha}_1(\hat{\alpha}_2)$  is downward (upward) biased in the limit.

Table I. Production function estimates

	$Q$ (S1) <sup>a</sup>	$Y$ (S2) <sup>a</sup>	$Y$ (S3) <sup>a</sup>	VA (S1) <sup>b</sup>	deVA (S2) <sup>b</sup>
Labour	0.324*** (0.068)	0.187*** (0.065)	0.192*** (0.065)	0.876*** (0.126)	0.685*** (0.133)
Materials	0.616*** (0.047)	0.669*** (0.050)	0.665*** (0.050)	—	—
Capital	-0.043 (0.063)	-0.118** (0.061)	-0.118** (0.061)	-0.078 (0.112)	-0.223** (0.118)
Ind. output	—	—	0.231*** (0.043)	—	—
Ind. dum.	Inc.	Inc.	Inc.	Inc.	Inc.
Time dum.	Inc.	Inc.	Inc.	Inc.	Inc.
Period	1990–99	1990–99	1990–99	1990–97	1990–97
No. obs.	11,476	11,476	11,476	8,687	8,687
Sargan $T$ (d.f.)	55.4 (55)	54.6 (43)	46.4 (43)	42.5 (42)	53.6 (34)
m1	-9.54	-9.56	-9.40	-10.28	-11.27
m2	-0.25	-0.60	-0.32	-1.32	-2.50

Estimation method: GMM estimates. Heteroskedasticity robust standard errors shown in parentheses.

Significant at: \* 10% level; \*\* 5%; \*\*\* 1%.

<sup>a</sup> IV  $\hat{s}$ : number of workers, physical capital and materials lagged levels from  $t - 2$  to  $t - 4$ ; (exogenous variables) growth of capital, growth in industry output (for specification S3).

<sup>b</sup> IV  $\hat{s}$ : number of workers from  $t - 2$  to  $t - 4$ ; physical capital from  $t$  to  $t - 3$ .

Note that  $\varepsilon^{\text{ob}}$  and  $\varepsilon^{\text{unob}}$  correspond to the scale elasticity of standard inputs when ‘quantity’ measure of output and materials are observable or unobservable, respectively. Moreover, the error term in (S2) picks up differences between changes in firm output and intermediate input prices and the corresponding industry price indexes,  $(\tilde{p}_{it} - \tilde{p}_{It})$  and  $(\tilde{g}_{it} - \tilde{g}_{It})$ . Section 4 shows estimation results when value added is used as an alternative dependent variable.

KG show that, in principle, the growth rate in industry output,  $\tilde{q}_{It}$ , ensures identification of demand elasticities and production function parameters (see equation (9) above). If this were the case, input elasticities obtained from estimating equation (S1) can be inferred, even when we use deflated revenues and intermediate inputs expenditure, by adding the term  $\tilde{q}_{It}$  to model (S2). This yields

$$\tilde{y}_{it} - \tilde{k}_{it} = \gamma_1(\tilde{l}_{it} - \tilde{k}_{it}) + \gamma_2(\tilde{n}_{it} - \tilde{k}_{it}) + (\varepsilon^{\text{unob}} - 1)\tilde{k}_{it} + \gamma_6\tilde{q}_{It} + \tilde{u}_{it}^{\text{unob}} \quad (\text{S3})$$

As mentioned in the introduction, the required predeterminedness of the right-hand variables to get consistent OLS estimates is unlikely to hold for some inputs of the production function. In particular, I assume that short-run factors of production, labour and materials, are possibly correlated with the error term. To avoid the ‘simultaneity bias’ due to the transmission of productivity shocks, I use GMM estimators with lags of labour and materials as instruments of the endogenous variables (see notes to Table I).

### 3. DATA AND VARIABLES

The data used in this study are retrieved from the *Encuesta sobre Estrategias Empresariales*, ESEE (Business Strategy Survey), an unbalanced panel sample of Spanish manufacturing firms published by the Fundación Empresa Pública covering the period 1990–1999. The raw dataset consists of 3151 firms for a total number of 18 680 observations. A ‘cleaned’ sample is obtained by applying the filters described in the Appendix. The sample employed here consists of all the firms that have been surveyed for at least 3 years after dropping all the time observations for which the

data required to the estimation are not available. Firms are classified into 15 different industries, listed in the Appendix. The Appendix also reports explanations of the variables used across the specifications stated in Section 2.2 together with descriptive statistics.<sup>17</sup>

A unique feature of the dataset is the availability of information on the price changes set by the firm together with price changes charged to the firm for its intermediate inputs. This allows me to construct a 'physical' measure of output,  $Q$ , and materials,  $M$ .<sup>18</sup> Subtracting the latter from the first, we can also compute the value-added, VA. Given that the firm-level price deflator reported in the dataset is a Paasche-type price index, a reliable measure of 'physical' output can be *a priori* computed for multiproduct firms, too. On an empirical ground, I check whether the estimates are affected by the presence of multiproduct firms by estimating the empirical specifications above only for the sample of uniproduct firms and I find that the results obtained are consistent with those reported in Section 4, where both types of firms are included in the sample.

Complementary information about price deflators for gross production, expenditure in materials and gross value-added are taken from the Spanish National Institute of Statistics (Instituto Nacional de Estadística, INE). The series of output deflators for gross production at 15-industry level cover all the period 1990–1999, while those for value-added are limited to the period 1990–1997. Using these series, two alternative dependent variables are defined: deflated revenues,  $Y$ , and deflated value added,  $deVA$ . Deflated expenditure in materials,  $N$ , is computed using an overall deflator for non-durable goods. These alternative measures of output and intermediate inputs allow one to assess the magnitude of the bias due to the use of general deflators. Labour inputs are measured by man hours, taking into account overtime and lost hours, so that this variable can adapt better to potential fluctuations due to the economic cycle. Capital,  $K$ , is computed recursively from an initial estimate (based on book values adjusted to take account of replacement values) and data on firms' investments in equipment goods  $I$ :  $K_{it} = (1 - \delta_j) * K_{it-1} + I_{it}$ . The subscript  $j$  denotes the use of sectorial estimates for the rate of depreciation,  $\delta$ . Real capital is then obtained using an overall investment deflator (for durable goods).

Finally, the growth rate in industry output,  $\tilde{q}_{it}$ , is estimated using the industry's weighted average of deflated revenues growth rates, with firms' average output shares in the two years as weights.<sup>19</sup>

#### 4. REGRESSION RESULTS

In this section, I assess the effects of using industry-wide deflators instead of firm-level data on prices. To this end, the number of observations and the set of instrumental variables are held fixed across the specification presented below so that any differences in estimates can be attributed to the change in the variables used. Table I presents the results obtained by estimating

<sup>17</sup> The unbalanced nature of the dataset is due to the fact that some firms exit the survey due to death and attrition. Given that firms' choices to liquidate are likely to depend on their productivity, the estimation of the production function may be affected by a selection problem. Olley and Pakes (1996) shows that more plausible estimates of the production function coefficients are obtained when taking into account this selection problem.

<sup>18</sup> Suppose that we have data on revenues for two consecutive years:  $P_{it} * Q_{it}$  and  $P_{it+1} * Q_{it+1}$ . Having access to data on price changes, we can express the figures above in terms of the reference year  $t$ :  $P_{it} * Q_{it}$  and  $P_{it} * Q_{it+1}$ . At this point, if we take log first-differences, we have a measure of the output growth rate,  $(\log(Q_{it+1}) - \log(Q_{it}))$ , free from price effects.

<sup>19</sup> This is the same approach used by KG.

the production function (S1) and (S2) as well as model (S3) where industry output is added as an extra regressor. The first interesting result is observed after comparing columns 1 and 2. Scale elasticities are higher when 'physical' output,  $Q$ , and materials,  $M$ , are used instead of deflated revenues,  $Y$ , and expenditure in intermediate inputs,  $N$ , with point estimates of 0.957 and 0.882, respectively. In particular, the null hypothesis of constant return to scale is rejected at the 5% significance level only in specification (S2). This result is consistent with the idea of downward bias in scale estimates advanced by KG. However, looking at the estimated coefficients of short-run inputs, it appears that only the coefficient of labour is downward biased (and largely so), while the coefficient of materials is quite stable across the two specifications. In Section 2, I conjecture that the downward bias due to the omitted output price can be either exacerbated or offset when there is also an omitted input price buried in the residuals. This result confirms that the practice of using common price deflators does not affect all of the inputs coefficients in the same way.<sup>20</sup>

All of these findings are confirmed using value added as the dependent variable. In column 4 the firm specific measure of value added, VA,<sup>21</sup> is used, while in column 5 the dependent variable, deVA, is computed using industry-wide deflators.<sup>22</sup> Estimated scale elasticities in these two columns are 0.922 and 0.776, respectively; that is, 15 percentage points lower when using industry-wide deflators. Once again, the null hypothesis of constant returns to scale is rejected only for this specification, mainly because of the large downward bias in the point estimate of the coefficient of labour.<sup>23</sup>

Column 3 shows the results of estimating specification (S3). The growth in industry output is highly significant. Interestingly, adding this industry variable to the model does not affect the estimated coefficients of the other variables. Moreover, the point estimate, 0.231, is very close to the results reported in Table II of KG (0.233). This value implies a demand elasticity,  $\eta$ , of  $-4.33 (= -\frac{1}{0.231})$  and a 'correction' term  $\frac{\eta}{1+\eta} = 1.30$ . This result gives only partial support to the KG solution. On one side, multiplying the estimated elasticity 0.882 by 1.30, we get a 'corrected' elasticity of 1.15: we can then reject the hypothesis of decreasing return to scale.<sup>24</sup> But, applying this correction factor to the single estimates of input coefficients does not lead to defining elasticities similar to those obtained using firm-level deflators. If the labour coefficient (the only parameter that is significantly affected by price deflators) in column 3 is multiplied for 1.30, we obtain a value of around 0.25, which is still lower than the corresponding value in column 1. At the same time, the coefficient of materials would take a value of 0.86 ( $= 0.66 * 1.30$ ), which is clearly well above the estimates generally obtained in empirical analysis of production functions. This casts some doubt on the validity of this approach.

It is important to check whether the differences between the estimated coefficients of the variables computed using firm-level deflators—specification (S1)—and those computed with

<sup>20</sup> Similar results (available upon request from the author) are obtained using the specification suggested by Hall (1988), where input coefficients are proxied by revenue shares, and estimates of (average) industry mark-ups can be obtained.

<sup>21</sup> This is defined as the difference between output,  $Q$ , and materials,  $M$ .

<sup>22</sup> As mentioned in Section 3, I find value-added deflators only for the period 1990–1997.

<sup>23</sup> In order to check to what extent my estimates are affected by the problem of quality changes, I have estimated specification (S1) and (S2) only for the subsample of five industries (numbers 1, 5, 7, 8 and 9) out of the 15 listed in the Appendix where R&D expenditures are higher and, therefore, where quality changes are likely to be more important. The results obtained (not reported here) confirmed the main findings of Table I in the paper. This seems to give further support to the idea that we can get reliable estimates of scale elasticities using 'physical' measure of inputs and outputs, even in the presence of quality changes.

<sup>24</sup> This result is of the same order of magnitude to those reported in Table VII of KG.

Table II. Test of differences in coefficients

Variables	Coefficient	Table I	
		$Q$ vs. $Y^a$	VA vs. deVA <sup>b</sup>
Labour	$\alpha_1 - \gamma_1$	0.117 [0.065; 0.166]	0.162 [0.029; 0.312]
Materials	$\alpha_2 - \gamma_2$	-0.044 [-0.082; -0.011]	
Capital (c.r. to s.)	$\varepsilon^{ob} - \varepsilon^{unob}$	0.065 [0.020; 0.106]	0.120 [0.008; 0.254]

Estimation method: GMM estimates. Values reported are the bootstrap average differences and, in brackets, the 90% confidence interval based on 400 replications of the GMM estimations.

<sup>a</sup> Differences between the coefficients of specification (S1) and specification (S2) in the first two columns of Table I.

<sup>b</sup> Differences between the coefficients of specification (S1) and specification (S2) in the last two columns of Table I.

industry-wide deflators—specification (S2)—are statistically different from zero. Table II shows formal significant tests on the differences between these coefficients using the method of bootstrapping: it reports the average differences of the estimated coefficients together with 90% bootstrapped confidence interval (in square brackets) based on 400 replications of the GMM estimates.<sup>25</sup> The first row shows that the coefficient of labour is sensibly higher (11 percentage points for output and 16 percentage points for value-added) when we use firm-level deflators in computing the variables of the production function. The estimated confidence intervals confirm that these differences are significantly different from zero. Results in the second row suggest that there is a small difference in the estimates of the coefficient of materials, which is a bit higher in the specifications with industry-wide deflators. Finally, the last row of results confirms that the scale elasticities  $\varepsilon$  are higher when we use firm-level deflators. The largest difference is found for the model with value-added as dependent variable. Here the average difference is 12 percentage points, with a confidence interval that goes from about 1 to 25 percentage points.<sup>26</sup>

## 5. CONCLUSIONS

This paper explores the reasons why GMM estimators of production function parameters are generally found to produce unreasonably low estimates for returns to scale. This well-known

<sup>25</sup> Note that this estimation is based on the 'block' bootstrapping procedure. This consists in randomly drawing a sample of firms and, for each drawn firm, all the yearly observations available are used; i.e., the observations of a given firm are kept together.

<sup>26</sup> The results presented above are based on models where deviations from constant returns to scale are measured explicitly. Therefore, the well-known problem of small capital coefficient is somehow hidden. Nevertheless, it is easy to observe that the capital coefficient takes low values across all the specifications, thus confirming the problem found with estimators in differences. As a possible solution to this puzzle, I have defined a 'short-run' capital variable computed by multiplying the capital stock  $K$  and the capacity utilization reported by the firm. The underlying idea is that estimates of capital coefficients can be improved if the information on relative factor usage is adequately appreciated (see Ornaghi, 2003, for further details). Point estimates of this 'short-run' capital coefficient are more than five times higher than those obtained with the standard capital  $K$  (results not reported here). Given the relevance of this issue, it is my intention to address it more extensively in a future paper.

finding is attributed to the inaccurate construction of the variables used in production function analysis. In particular, following the original insight of KG, I explore whether the problem lies in the use of common price deflators for revenues and expenditure in intermediate inputs.

The practice of using deflated revenues and deflated expenditure in materials instead of a 'quantity' measure of output and intermediate inputs is found to lead to lower scale estimates, mainly due to a relevant downward bias in the labour coefficient. Despite confirming the original KG critique, the results obtained show that their suggestion of using aggregate industry output,  $\tilde{q}_{It}$ , to infer the true values of input coefficients does not seem to provide an adequate solution.

Blundell and Bond (2000) find that more reasonable parameters estimates can be obtained using composed or extended GMM estimators, where lagged first differences are considered informative instruments for the endogenous variables in levels. This paper shows that great improvements in estimating the production function parameters can be derived from a more careful construction of the (dependent and independent) variables, besides the refinement of the econometric techniques.

There are some important questions that have not been addressed in this work. First, our results suggest that it is possible to get better estimates of input elasticities and returns to scale leaving the issue of quality changes aside. Nevertheless, it would be interesting to see whether a further improvement of these estimates can be obtained with quality adjusted prices (hedonic prices). Second, econometricians observe the aggregate quantity of output produced and inputs used over defined intervals, but they cannot observe firms' instantaneous production decisions. This poses the interesting question on whether (and how) we can make inferences about the parameters of a production function from time-aggregated annual data. By answering these questions, we can shed further lights on this important and still controversial research topic.

## APPENDIX

### Construction of Data Sample

The survey provides data on manufacturing firms with 10 or more employees. When this was designed, all firms with more than 200 employees were required to participate, while a representative sample of about 5% of the firms with 200 or fewer employees was randomly selected. In 1990, the first year of the panel, 715 firms with more than 200 employees were surveyed, which accounts for 68% of all the Spanish firms of this size. Newly established firms have been added every subsequent year to replace the exits due to death and attrition. The initial sample gathers 3151 firms in unbalanced panel data. The total number of observations is 18 680. I then clean our dataset according to the following criteria:

1. I remove all the observations where the log difference of the R&D variable between two consecutive years exceeds 4 in absolute value. This removes 64 observations.
2. I drop the observations where the log difference of the capital stock variable between two consecutive years exceeds 2 in absolute value. This removes another 81 observations.
3. For the specification using value-added, I need to remove the observations with negative value-added. There are 130 such observations.

The sample employed here results from retaining the firms with more than three consecutive observations, after removing all the time observations for which the data required to the estimation are not available. Tables presented in Section 4 report the exact number of observations making up the final samples. The original industrial classification reported in the survey is based on 18 sectors. A classification based on 15 sectors is used here since deflators for gross production were found at this level of aggregation. These 15 sectors are: (1) Ferrous and non ferrous metals; (2) Non-metallic minerals; (3) Chemical products; (4) Metal products; (5) Industrial and agricultural machinery; (6) Office and data-processing machine; (7) Electrical and electronic goods; (8) Vehicles, cars and motors; (9) Other transport equipment; (10) Food and beverages; (11) Textiles, clothing and shoes; (12) Timber and furniture; (13) Paper and printing; (14) Rubber and plastic products; (15) Other manufacturing products.

### Description of Variables

*Deflated gross production (Y)*: Gross production is defined as the sum of revenues and the variation of inventories. I deflate the nominal amount using output deflators for the 15 industries defined above.

*Deflated expenditure in materials (N)*: I deflate total expenditure in intermediate inputs as reported in the dataset using a deflator for non-durable goods.

*Deflated value-added (deVA)*: This has been computed subtracting total expenditure in intermediate inputs from the gross production and then using a deflator of value-added provided by the National Institute of Statistics for the 15 industries listed above.

*Labour (L)*: Labour consists of the total hours of work. It has been constructed using the number of workers, adjusted for the double counting of R&D employees, times the normal hours plus overtime and minus lost hours.

*Materials (M)*: Nominal materials are given by the sum of purchases and external services minus the variation of intermediate inventories. Nominal intermediate consumption is deflated by the firm's specific price index. This is a Paasche-type price index computed from the percentage variations in the cost of purchased materials, energy and services reported by the firm.

*Output (Q)*: Nominal output is defined as the sum of revenues and the variation of inventories. We deflate the nominal amount using the firm's specific output price as reported in the dataset.

Table III. Descriptive statistics of main variables

Variable	Name	Mean	SD
Output	$Q$	0.0369	0.279
Deflated gross prod.	$Y$	0.0282	0.279
Value-added	$VA$	0.0505	0.463
Deflated value-added	deVA	0.0018	0.502
Labour	$L$	0.0015	0.211
Materials	$M$	0.0266	0.382
Deflated expen. materials	$N$	0.0276	0.383
Physical capital	$K$	0.0775	0.248

Growth rates of the variable (logarithmic differences). Sample period: 1990–1999.

The firm-level price deflator is a Paasche-type index, computed from the price changes reported by the firm for each product that it sells on the market.

*Physical capital (K)*: This has been constructed capitalizing firms' investments in machinery and equipment and using sectorial rates of depreciation. The capital stock does not include buildings. This variable is taken from Martin and Suarez (1997).

*Value-added (VA)*: This is computed subtracting the value of materials,  $M$ , from output,  $Q$ . As both these variables are based on firm-level prices, we consider VA as our 'true' measure of production. Descriptive statistics of the variables are reported in Table III.

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